TRANSITION WAVES IN CONTROLLABLE CELLULAR STRUCTURES WITH HIGH STRUCTURAL RESISTANCE

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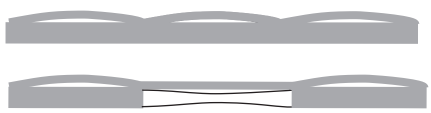
Summary The paper suggests an approach for optimization of morphology of mechanical structures subjected to an impact. We consider chains or lattices with breakable bistable links. A nonmonotonic constitutive relation for each link consists of two stable branches separated by an unstable branch. Mechanically, this model can be envisioned as a "twin-element" structure which consists of two elastic-brittle or elastic-plastic links (rods or strands) of different lengths joined by the ends. The longer link does not resist to the loading until the shorter rod breaks or develops a neck. When a chain or lattice of these elements is elongated they excite waves of damage that carry the energy away. We analytically describe and simulate transition waves of damage in bistable structures. We show that strength against an impact of the chain or lattice with nonmonotonic links increases several times; such structure can absorb much more energy before breakage than a conventional structure.

INTRODUCTION

The paper investigates the ways to increase structural resistance to a dynamic impact (collision). Partial damage of a material can be used for efficient energy absorption, while concentrated damage leads to failure. A structure that absorbs the kinetic energy of a collision necessarily experiences some damage – an irreversible change in the material properties. Theoretically, a material can absorb energy until it melts but the real structures are destroyed by a tiny fraction of this energy due to material instabilities such as cracks and necks; after the whole structure fails, most of its material remains in a fairly good shape. Therefore, optimal design of a structure under collision can be viewed as a problem of design of a structure that distributes a moderate damage over a large volume and excites high-frequency waves which transform the energy to the heat. We suggest to increase the dynamic strength of structures by using elements of special morphology – damageable “waiting links” which are characterized by nonmonotonic force–elongation dependence.

WAVES IN A BREAKABLE CHAIN

As an example, we consider irreversible locally unstable “waiting links” that dissipate the energy by distributing “partial damage." The cellular element of the structure contains two roughly parallel rods, one of which (the longer one) is initially inactive and starts to resist only when the strain is large enough and when the shorter rod is broken of damaged (see Figure 1, left). Networks of "waiting links" exhibit phase transition and excite intensive elastic waves; multiple breakages absorb the energy. The transition is of the type of mild explosion: The energy stored in a bistable link is released when the wave hits the link.



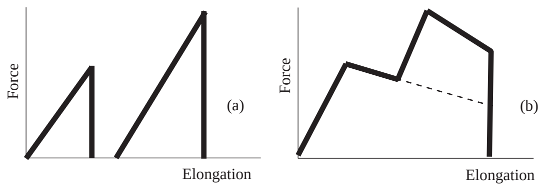


Figure 1. Left: Chain with waiting links. Center and right: The constitutive relation in the waiting link: Elastic-brittle material and elastic-plastic material, respectively.

Our model of a cellular chain with locally unstable links is described by the equation

$$m\ddot{x_k} = F(x_{k+1} - x_k - a, t) - F(x_k - x_{k-1} - a, t), \quad k = -\infty, \dots, \infty \tag{1}$$

where x_k is the coordinate of the kth node, a is the distance between masses at rest, a nonmonotonic (and may be discontinuous) function F is the tensile force in the locally unstable link and m is the dimensionless mass. The tensile force in the link F is a composition of the forces in the two rods which are determined from material constitutive relations $f(z, t)$,

$$F(z, t) = \alpha f(z, t) + (1 - \alpha) f(z - \tau, t)$$

where α is the relative thickness of the shorter link and τ is the delay parameter reflecting the difference in length of the links. The tensile force $F(z, t)$ for a monotonic elongation z of breakable brittle-elastic material and elastic-plastic material is shown in Figure 1 (center and right); the dependence on t (not shown here) accounts for irreversible character of the damage.

The wave of damage in the chain with piece-wise linear unstable links can be described as a wave in an equivalent linear chain subjected to running forces that model the transition. With the assumption that a wave of transition propagates with a constant unknown speed c, $x_k(t) = x_{k+1}(t + a/c) + a$, the infinite system (1) can be reduced to a single equation which, in the case $\alpha = .5$, has the form

$$m\ddot{x} = E\left(x(t + a/c) - 2x(t) + x(t - a/c)\right) + P(t), \tag{2}$$

where x is the coordinate of the mass at the wave front, E is the stiffness, and $P(t)$ is the time-dependent force that compensates the drop in tensile forces due to the breakage. This force can be calculated from the model: In the case of elastic-brittle material, it is a combination of two step functions, $P(t) = H(t) - H(t + a/c)$, and in the case of elastic-plastic material, it is a piece-wise linear function. Subsequently, the Fourier transform and the Wiener-Hopf technique (required in the case when the stable branches are not parallel, $\alpha \neq .5$) are used to integrate the obtained equation (2)

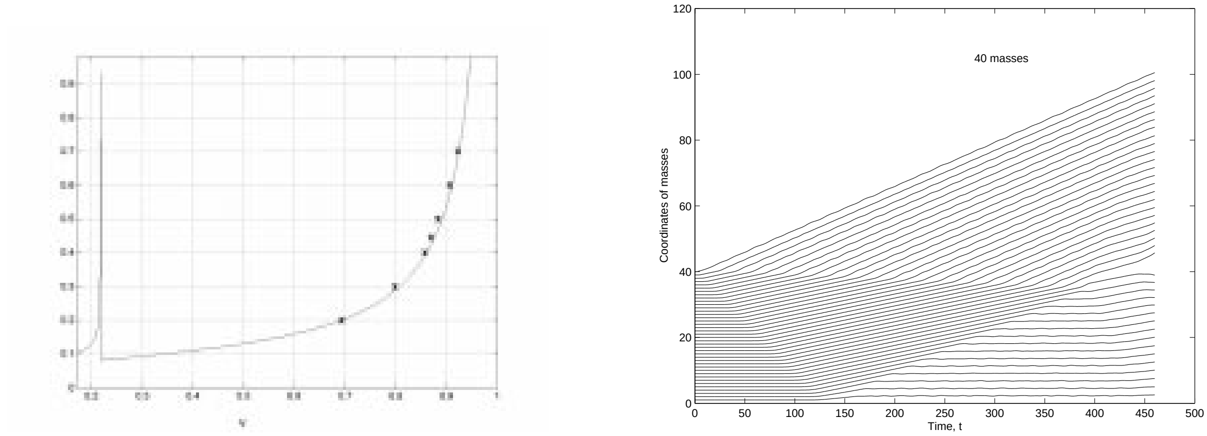


Figure 2. Left: The calculated dependence of the speed of the wave of transition on the prestress. Dots show the result of simulation. Right: The wave of transition in an elastic-plastic chain.

We derive the conditions for the wave of partial damage to propagate through the structure (the "house of cards problem") and describe the waves of transitions in these two models. Particularly, we obtain the conditions of existence of the transition wave, evaluate the energy consumption, and find the speed of the transition wave as a function of external force applied to the chain (Figure 2, left).

OPTIMIZATION OF DESIGN AND DEVELOPMENTS

The structures with waiting links distribute the partial damage along their volumes instead of localizing it as the conventional structures do; the energy absorbed by these structures is several times greater. Even larger effect is achieved when the structures are specially designed: The material is optimally distributed between the links. The optimization of the finite chain is based on the anticipated wave propagation; for the dynamical process shown in Figure 2, right, the links at the ends of the chain where the damage wave starts should be reinforced as well as the links in the region where the damage waves from both sides of the chain meet. The design reflects the whole history of the wave propagation.

Using a similar model for the two-dimensional networks, we find that they possess a long-range ordering that causes propagation of the partial damage. The partial damage of a link in the lattice causes stress concentration in neighboring links; these links experience the phase transition as well, and a partial damage is spread like a crack. In the limit, the model allows to formulate dynamic equation of material continuum that takes into account the energy of high-frequency modes, the speed of the phase transition, and self-strain derived from the discrete model.

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