Bounds for Detectability of Material Damage by Noisy Electrical Measurements

Andrej Cherkaev & **Elena Cherkaeva**
Department of Mathematics **Department of Geophysics**
University of Utah, Salt Lake City, UT 84112, U.S.A.

A problem of detection of a damage of material by nondestructive electrical measurements is considered. The addressed question is whether or not a conducting body contains damaged regions inside? Only a potential presence of damaged material is of interest, not a location of damaged regions, their shape, etc. The problem of detection naturally arises in technical diagnostics, where the presence of cracks, corrosion, or any other defects in a sample is to be determined.

To detect a damage we can apply arbitrary currents on the surface of the body and measure the energy required to inject these currents. The detection is based on comparing energies spent to inject the same current in an undamaged body (which we call a model body) and in a real sample.

It it assumed that a damaged material differs by its conductivity from a basic material. Damaged parts of the sample are called inclusions. It is supposed for definiteness that the conductivity of inclusions is less than the conductivity of the basic material; then the energy spent to inject a current into a body with inclusions is greater than the energy for the same body without inclusion. The following assumption makes the problem nontrivial: the sensitivity (the resolution) of measurements is limited. An inclusion is detected if the difference between the spent energies for the body with and without inclusions is greater then the sensitivity of measurements.

Nothing supposed to be known about inclusions location; we deal with the problem of finding an injected current which detects the 'worst' position of inclusions. So we solve here the following hide and seek game: the inclusion hides itself and the injected current detects its presence. We determine the best strategies for both players; one of them controls the position and shape of inclusions and the other controls the injected currents. The abilities of both sides are restricted by fixing the total volume of inclusions and the total energy of injected currents.

Formally the problem is formulated as an optimal spectral problem. The detectability of a fixed system of inclusions is equal to the maximal eigenvalue of a variational problem for applied boundary currents, and the optimal injected current is the eigenfunction of this problem. 'The worst' location of inclusions corresponds to minimization of this eigenvalue. We observe that formally the problem is close to the known problem of maximization of the principle eigenfrequence of vibrating construction (Olhoff, Rasmussen, Bratus, Seyranian, Cox, Overton). Although the energy has a different form the problems have common features, including multimodality.

The necessary conditions of optimality show, in particular, that the best policy is to provide a current of uniform density within the model body; the current's direction is arbitrary. The most hidden inclusions are laminates along the applied current and uniformly distributed.

Moreover, the problem does not possess a saddle point solution: the cost of the problem depends on the sequence of players turns. We reformulate this game problem using mixed strategies to find the best policy in the series of experiments. Solution of reformulated problem shows that the optimal strategy is to inject several orthogonal currents with fixed total energy rather than one current with the same energy. The best strategy for inclusions is to be dispersed into infinite small pieces which are distributed uniformly throughout the body; the optimal inclusions form an isotropic composite material! The theory of the composite materials with extremal properties is used to explicitly calculate the detectability, to find the best shapes of inclusions, etc.

Solving the problem we find a priori bounds for sensitivity of measurements which is required for detecting inclusions no matter how they are located. Simultaneously, the dual problem is solved: what minimal amount of arbitrary located damaged material can be detected by measurements of given sensitivity. The bounds have an explicit and simple analytical form which makes it convenient for use.