Optimal Design of Three-Dimensional Axisymmetric Elastic Structures

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The problem of maximizing the the overall stiffness of an elastic axisymmetric body comprised of given materials is treated. Particular examples include the problem of topology optimization: the optimal shape and structure of shells, plates, domes, cantilevers, etc. is found. The axisymmetric problem has all the main features of general three-dimensional elasticity problems: the microstructures are arbitrary (fully three-dimensional), the stress tensor has arbitrary eigenvalues, etc. On the other hand, the axial symmetry of the stresses reduces the number of degrees of freedom in the numerical procedures to a level which is more tractable. It still requires the development of efficient algorithms for a converging sequence of ill-conditioned elastic equilibrium problems which we will discuss, that may be applied to fully three-dimensional problems. The axisymmetric setting is also convenient for graphical display of optimal out-of-plane examples.

Here we combine recently developed variational methods of optimization of three dimensional composite structures with numerical algorithms to find optimal projects. To our knowledge, the computations we have already made are the first rigorously optimized out-of-plane elastic bodies yet computed, and will provide useful comparisons for more general three-dimensional computations in the future.

Particularly, we compute the shape of optimal circular plate and of optimal cylindrical bar under torsion in three-dimensional settings, and compare our solutions with ones obtained by earlier authors who have used two dimensional models. Many results are dramatically different.