LAB 8 - STABILITY OF THE RICKER MODEL

MATH 1170

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In this lab, we'll continue exploring derivatives.

- study the equilibria of a dynamical system
- understand the stability properties of the equilibria
- use the Ricker model as a case study

History

In class, you will study the logistic growth model,a fairly simple population model. However, due to its simplicity, it has its limitations. [1] Specifically, a major problem with the logistic growth model is that large populations in the model return a negative population in the next generation, which is clearly unrealistic. Ideally, we would like the updating function remains positive.

In the 1950s, Dr. Bill Ricker (seen to the right, being awesome)

a Canadian fishery biologist sought to produce a more realistic model of salmon populations. Due to the success of this work and many other huge contributions to the field, Ricker is now regarded as Canada's greatest fisheries biologist. [2] In fact, the Ricker model is still used today all throughout the world to predict fish populations.

[1] this does not mean it is useless

[2] according to `http://www.science.ca/scientists/scientistprofile.php?pID=17`

Ricker Model

Similarly to the logistic model, the Ricker model is a discrete dynamical system, which gives the expected number (or density) of salmon x_{t+1} in generation $t+1$ as a function of the number of salmon in the previous generation. The model is summarized by

$$x_{t+1} = r x_t e^{-x_t}, \tag{1}$$

where $r > 0$ is a constant that describes the growth rate. In this lab, we'll explore the equilibria (and their stability) and appreciate the beauty of the Ricker model.

Question 1:

By hand, find the equilibria of the model. Show your work. *Hint: You should find two equilibria.*

Question 2:

Using the results from your previous question, how large must r be for there to be a an equilibrium that makes sense? Your answer to this question should take this form: *We need $r >$ __ for there to be a positive equilibrium*, where you fill in __. Explain your answer.

Stability in the Ricker model

Question 3:

Calculate the non-negative equilibria and their stability of the Ricker model for different values of r. The calculation can be done either by hand or coding (either way, show the complete work for full credit). Then fill in the following chart. Use the stability theorem to evaluate the stability at each of the equilibria. [3]

r	x^*	$f'(x^*)$	Stability
0.5			
5.0			
5.0			
9.0			
9.0			
13.0			
13.0			

Note that I've included two copies in the places that you should have two equilibria.[4]

[3] Remember the stability theorem for discrete-time dynamical systems:
Theorem. Suppose that the discrete-time dynamical system $x_{t+1} = f(x_t)$ has an equilibrium at x^*. Let $f'(x)$ be the derivative with respect to x. The equilibrium at x^* is stable if

$$|f'(x^*)| < 1$$

and unstable if

$$|f'(x^*)| > 1.$$

[4] as a sanity check for your previous work

Question 4:

Using the provided code `cobweb_ricker.R`, you should create 4 cobweb diagrams: one for $r = 0.5$, one for $r = 5$, one for $r = 9$, and one for $r = 13$.

Hint: when you change the r value for different plots, remember to 'save' the updated code, before 'source' it. Also, you can play around with the initial point `X0` but we're not so worried about this yet.

Question 5:

You may have noticed that for some values of r, the population oscillates - the number of fish overshoots the equilibrium, then it undershoots the next year, and so on. Can you think of a biological explanation for why this might happen in a salmon population?