LAB 6 - DERIVATIVES CONT'D

MATH 1170

25 SEPTEMBER 2018

In this lab, we'll continue exploring derivatives.

- use R to symbolically compute derivatives for us
- take the derivative of a series
- approximate a function by some other functions

Symbolic derivatives

In previous labs, we used R to **approximate**[1] the derivative of a function at various points. R can [2] also compute derivatives symbolically, which means it can tell us the expression for the derivative of a given function.

[1] because a computer doesn't know what $\lim_{h \to 0}$ means

[2] does not always mean *should*

Consider some function, say

$$f(x) = x^3.$$

First, think in your brain what the derivative of this function should be. To have R compute the derivative, we need to tell R what the function is, which we do by

the syntax here is: (your function, and then the variable in quotes)

```
>  f <- expression(x^3,'x')
```

We use the new command `D` to find the expression for the derivative.[3]

[3] `D` takes two arguments: the expression and the differentiation variable, exactly what we defined! how convenient!

Use this command by

```
>  dfdx <- D(f,'x')
```

Here, we gave the derivative the name [4] `dfdx` but you can call it whatever you would like!

[4] because this expression is $\frac{\mathrm{d}f}{\mathrm{d}x}$ but hopefully this is obvious

To see the fruits of our labor, type

```
>  print(dfdx)
```

R should print out

```
3*x^2
```

This is hopefully be exactly what we expected!

Question 0:

Although computers are quite stupid, this means they excel at stupid, mechanical tasks like taking a derivative. Try having R compute the derivative of some crazy function like

$$f(x) = \log(3x^2 + x + 4).$$

Question 1:

1. What is the derivative of $f(x) = 2^x$?
2. What is the derivative of the derivative of $f(x)$ (i.e., the second derivative)?
3. What is the derivative of the derivative of the derivative of $f(x)$ (i.e., the third derivative)?
4. What do you think the 47th derivative of $f(x)$ is?

Question 2:

Writing your own code, create a plot of $f(x)$ from **Question 1** with its first, second, and third derivatives. Put all of these on the same figure, in different colors.[5]

As a hint: Do something like the following to "bind" the y values. You may need to adapt this to plot your functions if you gave them a different name.

```
> x <- seq(0, 1, 0.1)
> f <- function(x){2^x}
> matplot(x,cbind(f(x), eval(dfdx), eval(dfdx2)),type="l",
   col=c("blue","red",'green','yellow'),ann="false")
```

where I'm assuming that you named your derivatives previously `dfdx`, `dfdx2` and so on. **Also, make sure this is all in one line in your code!**

[5] shades of gray are often a nice choice, where the command is `gray(c)`, where `c` is a number between 0 and 1.

A Series of Functions

Consider[6] the following list of polynomials:

$$
\begin{aligned}
p_0(x) &= 1 \\
p_1(x) &= 1 + x \\
p_2(x) &= 1 + x + \frac{x^2}{2} \\
p_3(x) &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \\
p_4(x) &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}
\end{aligned}
$$

[6] completely out of context. don't feel panicked if this seems out of nowhere

Question 3:

Hopefully there should be a pattern to the list. What do you think $p_5(x)$ and $p_6(x)$ are?

Question 4:

Plot the **SUM** of these functions. That is, plot

$$f(x) = p_0(x) + p_1(x) + \cdots + p_6(x).$$

What function[7] does this look like?

[7] it's a famous function!

Question 5:

Take the derivative of the **sum** $f(x)$ (*by hand*, or spend an eternity retyping these as `expressions` to use the `D` command).

Do you notice a pattern? Plot $f(x)$ and its derivative, $f'(x)$.

Question 6:

If the **sum** $f(x)$ is approximately the function you think it is from Question 4, why does its derivative make sense? That is, what is the derivative of the function you think it is?[8]

[8] in case this question is a complete dumpster fire, the function I have in mind is e^x