LAB 5 - INTRO DERIVATIVES

MATH 1170

18 SEPTEMBER 2018

In this lab, we'll be begin to explore derivatives. Specifically, we'll

- visualize the derivative as $h \to 0$
- plot a function and its derivative

Visualizing the Derivative as a Slope

Question 1

Suppose the number of dogs you own satisfies the equation

$$f(t) = e^t + 1. \tag{1}$$

Plot this function over the t range 0 to 2, where t represents the years from current. You should define f as a function called `f` in R.

Question 2

Recall that we can compute the slope of the **secant** line of $f(t)$ between the points at $t = a$ and $t = b$ by

$$\text{slope} = m = \frac{f(b) - f(a)}{b - a}.$$

This is nice, but not the form we need. If we call $b = a + h$ that is, b is a little tiny bit h off from a, then we can rewrite this as [1]

[1] this should hopefully look familiar

$$m = \frac{f(a+h) - f(a)}{h}. \tag{2}$$

If we're thinking of the line going through $(a, f(a))$ with the slope above, then we can write the equation of the line as[2]

[2] using point-slope form, do you remember this?

$$y - f(a) = m(x - a). \tag{3}$$

We'll compute the line through the point $a = 1$. Define this and $h = 1$. Using (2), compute the slope of this line. The intercept comes from (3), and using these two, we can plot the line, using the commands

```
> m <- (f(a+h)-f(a))/h;
> intercept <- f(a)-a*m;

> abline(intercept,m, col='red', lty=2  )
> points(c(a, a+h), c(f(a), f(a+h)), type='o', col='red')
```

Repeat this for points that are closer and closer. That is, take[3]

[3] and keep going if the pattern isn't clear

1. $h = .1$
2. $h = .01$
3. $h = .001$

Explain what's going on as $h \to 0$.

Question 3

What does the slope of this line[4] represent (in words)?

[4] this is exactly the derivative $f'(1)$!

Plotting Derivatives

Question 4

We[5] can think of the derivative as a *function*. That is, $g'(x)$ is a thing you put an x into and it spits out a slope defined by

[5] including you!

$$g'(x) \quad \stackrel{\text{def}}{=} \quad \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}. \tag{4}$$

Therefore, for any $g(x)$ we can associate its derivative $g'(x)$. For this probem, take

$$g(x) = 7x^4 + 28x - 6 \tag{5}$$

On the site, there is an **UNFINISHED** code called `lab5_plotderiv.r`. There are two key things missing:

1. defining the function $g(x)$
2. defining the derivative using (4) using the x and h values already assigned. Call this vector of numbers `slope`

Modify the code to do the two things listed above and show the resulting plot.

Question 5

From your plot from previous question, what does it mean when the derivative is negative? Positive? Zero?