LAB 4 - OXYGEN MODEL REVIEW

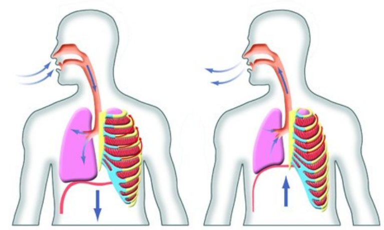
MATH 1170

12 SEPTEMBER 2018

In this lab, we'll review! Specifically, we'll perform some additional exploration of the discrete dynamical system model of oxygen formulated in class.

Absorption of Oxygen by the Lungs

Similar to medication, breathing creates a discrete-time dynamical system. Assume we want to track oxygen concentration inside the lung after each breath, t, where, $t = 1, 2, 3, \ldots$. Let c_t be the oxygen concentration in the lung before breathing and c_{t+1} is the oxygen concentration after breathing. Suppose the average adult lung has a volume V of about 6 L. With each breath, $W = 0.5$ L of air within the lung is exhaled and replaced by ambient (outside) air. Moreover, when we breathe, oxygen is absorbed via our lungs into the bloodstream. We want to investigate how different parameters will affect a discrete-time dynamical system lung model.



Suppose that a fraction q of the air in the lung is exchanged for each breath, that ambient air has a concentration γ. At the same time, some fraction α of oxygen in the lungs is absorbed before breathing out. The absorption model is as follows:[1]

$$c_{t+1} = (1-q)(1-\alpha)c_t + q\gamma.$$

[1] it's worth noting that this is slightly different than the version you derived in class. The only real difference is that $1-\alpha$ shows up only in one term because we're assuming that the old oxygen is absorbed *before* we inhale new oxygen

Question 1:

Assume that the ambient concentration of oxygen is 21%, and that 30% of the oxygen in the lungs is absorbed with each breath. Using this information, select the correct parameter values for α, γ, and q and write down the resulting discrete-time dynamical system.

Question 2:

Using the discrete-time dynamical system you wrote down in Question 1, write an R code, using 'For Loops', that will tell you the concentration of oxygen in the lungs after 100 breaths, with initially empty lungs.

Question 3:

Use the provided code titled `oxygen_cobweb.r` to create a cobwebbing diagram of this dynamical system with $\alpha = 0.3$ for 100 steps. Then,[2] run the code by typing

[2] after chaning your working directory

```
> source('oxygen_cobweb.r')
> cobweb(0.3,100)
```

The function `cobweb` takes two arguments. The first represents the value of α, and the second is the number of steps. What is the equilibrium concentration of oxygen when $\alpha = 0.3$?

Question 4:

Given $q = 0.1$ and $\gamma = 0.21$, find (mathematically[3]) the equilibrium concentration of oxygen as a function of α. Next, find the total oxygen absorbed per breath as a function of α (remember, total amount = volume $\cdot$ concentration). Plot both equilibrium concentration as a function of α and total absorbed oxygen as a function of α.

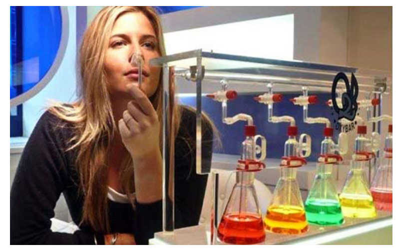
[3] or really through any means

Question 5:

Oxygen bars[4]

are equipped with tanks of 99% pure O2 for customers to breathe in through small tubes hooked over the ears and under the nose. While prolonged exposure to this level of oxygen is dangerous, proponents claim that short-term exposure creates a clarity of mind, alertness, a feeling of enhanced perception and high level of energy. A customer may pay $10 for 10 minutes, or $20 for 20 minutes. Assume that you have a respiration rate of 15 breaths per minute. If you want to spend at least 5 minutes at equilibrium oxygen concentration levels, do you need to spend $20, or will $10 suffice? [5]

[4] this is apparently a real thing



[5] as a hint: modify your code from **Question 2** to answer this

Question 6:

This question is completely unrelated to this lab, but hopefully cool. To "build" a function, sines and cosines are often used as the fundamental building block.[6] In this problem, we'll build a possibly unexpected function.

[6] this idea is called a *Fourier series* in case you've heard these words

1. Consider function $f_1(x) = \cos(x - \frac{\pi}{2})$ for x in the range $(-1, 1)$.
   (a) What is the period of this function?
   (b) Amplitude? Phase? Average?
   (c) Plot this function in R.
2. Repeat the same for $f_3(x) = \frac{\cos(3x - \frac{\pi}{2})}{3}$.
3. Next, just consider[7] the functions $f_5(x) = \frac{\cos(5x - \frac{\pi}{2})}{5}$, and $f_7(x) = \frac{\cos(7x - \frac{\pi}{2})}{7}$.
   (a) Plot the sum $f_1(x) + f_3(x)$.
   (b) Plot the sum $f_1(x) + f_3(x) + f_5(x)$.
   (c) You guessed it: plot the sum $f_1(x) + f_3(x) + f_5(x) + f_7(x)$.
   (d) What pattern do you see? If we kept going with f_9, f_{11} and so on, what do you think would happen? You're encouraged to try this!

[7] don't tell me all the same stuff about these