

# LAB 12 - 2D DYNAMICAL SYSTEMS

MATH 1170

NOVEMBER 27, 2018

In this lab, we'll explore a 2D dynamical system. Specifically, we'll

1. explore a classic love story in terms of a 2D system of differential equations
2. plot solutions in time and in the phase-plane
3. explore different types of exotic equilibria in higher dimensions

## Setup

This is an example dynamical system originally presented by Steven Strogatz<sup>1</sup> to illustrate differential equations in higher dimensions.

The power of mathematics has rarely been applied to the dynamics of romance, due to obvious difficulty in any model of love is defining what is meant by love and quantifying it in some meaningful way. However, I hope you'll find this fun and a reasonable way of approximating this.

Define  $R(t)$  to be Romeo's love<sup>2</sup> for Juliet at time  $t$  and define  $J(t)$  as Juliet's love for romeo. The model is then

$$\begin{aligned}\frac{dR}{dt} &= aJ \\ \frac{dJ}{dt} &= bR.\end{aligned}$$

What does this system say? Romeo and Juliet are both unable to listen to their own feelings,<sup>3</sup> but able to respond to the feelings of the other. That is,  $a$  represents how much Romeo's feelings are influenced by Juliet and  $b$  represents how much Juliet's feelings are influenced by Romeo.

### Question 1

Consider  $a = 2, b = 1$  by make this change in the file `lab14_romeojuliet_sim.r` and then choose an initial condition. First, try setting

$$R(0) = 1, \quad J(0) = 1.$$

That is, both Romeo and Juliet start with fond feelings of each other.

Does the romance work out in this case?

### Question 2

Again, try  $a = 2, b = 1$  with Juliet initially having slightly negative feelings<sup>4</sup> toward Romeo, so

$$R(0) = 2, \quad J(0) = -1.$$



<sup>1</sup> very entertaining mathematician. Both his books *Sync* and *the Joy of X* are recommended reads!

<sup>2</sup> or hate if negative

<sup>3</sup> because they're teenagers

<sup>4</sup> this is probably why this isn't usually math modeled. What does this even mean?

Does the romance still work out?

### Question 3

From the previous questions, you might think young love is infallible, but now, try Juliet having an even worse initial impression of Romeo, but him still being fond of her, so

$$J(0) = -2, \quad R(0) = 2.$$

### Question 4

We're going to understand this behavior by plotting a *phase portrait*, but before we do that, we need to install an R package to allow the code to work. In your R terminal, type

```
install.packages('phaseR')
```

and hopefully everything should go smoothly.

Now, after setting  $a = 2$  and  $b = 1$ , generate a phase portrait by running `lab14_romeojuliet_phase.r`. When you source this file, you get the option to click three starting locations to see where they go.<sup>5</sup>

Interpret the results to your previous questions using this generated plot.

### Question 5

Now, consider  $a = -2$  but  $b = 1$ . That is, Juliet still responds to Romeo's feelings appropriately but Romeo rejects Juliet's responses.<sup>6</sup> Start with say,

$$J(0) = 1, \quad R(0) = 1$$

to give their relationship the best chance of succeeding with positive initial impressions. What happens to `lab14_romeojuliet_sim.r` now?<sup>7</sup>

### Question 6

Again run `lab14_romeojuliet_phase.r` now with  $a = -2$  but  $b = 1$ .

Is there any initial impression that would cause this romance to succeed?

<sup>5</sup> the blue and green lines are what are called *nullclines* and are when  $dJ/dt = 0$  (in blue) and  $dR/dt = 0$  (in green). Their intersection are the equilibria

<sup>6</sup> this is often called the "narcissist" Romeo

<sup>7</sup> you may want to increase `tend=10` to see what happens over a wider interval