LAB 12 - DIFFERENTIAL EQUATIONS

MATH 1170

OCTOBER 30, 2017

In this lab, we'll explore differential equations a bit more in the context of a Newton's Law of cooling problem. Specifically, we'll

1. verify solutions to a differential equation
2. understand long term behavior of the solution
3. use Euler's method to approximate a solution

Setup

In this problem, we'll model the temperature of the temperature of the turkey and try to figure out if it will be too cold to bring to a Thanksgiving party.

We'll assume the temperature of the turkey (which we'll call P) at t minutes after taking it out of the oven can be modeled by **Newton's Law of Cooling**, which says, the temperature P satisfies

$$\frac{dP}{dt} = \alpha(A - P),$$

where A is the ambient temperature, which we'll assume is a brisk 50 degrees Fahrenheit and α is a parameter relating to how quickly the turkey cools[1], which we'll say is $\alpha = 0.1$.

[1] often called a rate constant. note it has units 1/time, also known as a rate

Question 1 As with most differential equations in the class, an initial condition is necessary. Somewhere in the previous information I've hidden the appropriate initial condition, so what is $P(0)$?

Question 2 Although we won't solve the differential equation yet, is dP/dt positive or negative? That is, is $P > A$ or $P < A$? Why does this make sense?

Question 3

Verify that the following is indeed a solution to our differential equation

$$P(t) = 350e^{-0.1t} + 50.$$

Question 4

Now that we're sure[2] the previous question gave us a solution to the differential equation, we can learn things from it.

[2] or at least pretty sure

What happens to the temperature of the turkey as $t \to \infty$? Why does this make sense?

Make a plot of the function that demonstrates this behavior.

Euler's Method

Although a previous part gave the solution to this differential equation, its not always the case that we can write something like this down and therefore want a way of approximating this. One way of doing so is Euler's method. Here I'll give a brief reminder of what Euler's method says.

Suppose we have some differential equation that looks like

$$\frac{\mathrm{d}P}{\mathrm{d}t} = f(P), \qquad P(0) = P_0.$$

We can then use a tangent line approximation to the true solution $P(t)$ to approximate the value at some time close to our initial point. Say, our timesteps are of size Δt, then the first point we approximate would then be[3]

$$P(\Delta t) \approx \hat{P}(\Delta t) = P(0) + P'(0)\Delta t,$$

but note that we know $P' = f$, so we know everything in this equation! How exciting. Repeating this procedure, calling $t_{new} = t_{old} + \Delta t$, we can then update using the same tangent line idea

$$\hat{P}_{new} = \hat{P}_{old} + P'(\hat{P}_{old})\Delta t.$$

Question 5
Complete the code `lab12_eulers.r` and show the resulting produced by utilizing Euler's method for this problem. Take timesteps of $dt = 5$ minutes and go until $t = 30$ minutes. How good of an approximation is this?

Question 6 Try a smaller value of dt of your choice. Does the approximation get better or worse? Why?

Question 7 Complete question 32 in section 4.2 in your textbook.

[3] convince yourself this is *exactly* the tangent line approximation at $t = 0$