

# LAB 10 - RIEMANN SUMS

MATH 1170

NOVEMBER 13 2018

In this lab, we'll use Riemann sums - both left and right endpoint sums - to approximate antiderivatives

## *Riemann Sums - Motivation*

For most<sup>1</sup> functions we cannot find antiderivatives, can't write down a nice way of evaluating the integral.

Fortunately another interpretation of a definite integral is the area between the function and the x-axis, commonly referred to as the "area under a curve". In order to approximate this area, we can divide the domain up into many intervals, for which we'll use an easy shape (rectangle) to approximate the area.

<sup>1</sup> clarification: write some random function down. you probably can't find an antiderivative

## *Left Endpoint Sum*

Suppose that we have a differential equation that we cannot solve explicitly. For example, in this lab, we will consider

$$\frac{dF}{dt} = e^{\sin(t)}, \quad F(0) = 0.$$

What is  $f$ ? It is a function whose derivative is  $e^{\sin(t)}$ , or in other words,  $f$  is the antiderivative of  $e^{\sin(t)}$ . Suppose that we want to solve this differential equation from  $t = 0 \dots 10$ . Then, another interpretation of our problem is integrating  $\frac{df}{dt}$  from  $t = 0$  to  $t = 10$ , i.e.

$$\int_0^{10} e^{\sin(t)} dt = F(10) - F(0) = F(10).$$

Unfortunately, since we don't know what the antiderivative  $f$  is, we will need to approximate it.

In order to approximate the integral in Equation (??) with the left endpoint sum, we need to first divide the domain  $t = [0, 10]$  into  $n$  equal subintervals. The length of each subinterval is

$$\Delta t = \frac{t_{max} - t_{min}}{n}.$$

We want to approximate the area under curve  $e^{\sin(t)}$  on each subinterval with the area of a rectangle (since that's easy for us to calculate).

We now know the width of the rectangle, which is the length of the

subinterval. Now we need to calculate the height of the rectangle. For now, let's choose the height of the rectangle to be the value of the curve  $e^{\sin(t)}$  at the **left endpoint** side of the interval, which is  $t = 0$ . That means the area of the first rectangle is

$$\text{width} \cdot \text{height} = \Delta t \cdot e^{\sin(0)} = \Delta t$$

Now, we continue this process for each subinterval. The second rectangle will have the same width, but the height will now be evaluated at the left endpoint side of the second subinterval, which is  $t = \Delta t$ . The area of the second rectangle is

$$\text{width} \cdot \text{height} = \Delta t \cdot e^{\sin(\Delta t)}$$

Similarly, the third rectangle has area

$$\text{width} \cdot \text{height} = \Delta t \cdot e^{\sin(2\Delta t)}$$

We can continue this process until we get to the end of all of the intervals. Our antiderivative  $f$  is approximated by adding the area of all these rectangles together. Mathematically, we can write this as

$$F(10) \approx \sum_{i=0}^{n-1} \Delta t \cdot e^{\sin(t_i)}$$

The **Right Endpoint Sum** rule is very similar, but instead of calculating the height of the rectangle as the value of the curve on the left side of the subinterval, we use the value of the curve on the right endpoint side of the subinterval. Adding the area of all these rectangles together yields

$$F(10) \approx \sum_{i=1}^n \Delta t \cdot e^{\sin(t_i)}$$

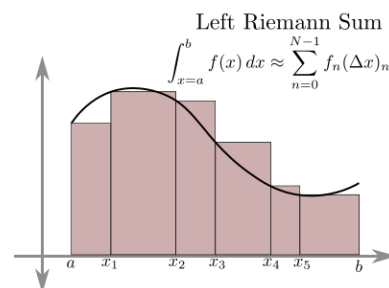
### Question 1:

By Hand, use the left endpoint sum to calculate an approximation to the integral

$$F(10) = \int_0^{10} e^{\sin(t)} dt,$$

using  $n = 5$  subintervals. Show your work by

- showing the calculation for the area of each subinterval,
- summing them together,
- use R to calculate your sum, which involves the evaluation of  $e^{\sin(\#)} + e^{\sin(\#)} + \dots$



**Question 2:**

Complete the provided code `lab11.R` to calculate antiderivative

$$F(10) = \int_0^{10} e^{\sin(t)} dt,$$

using the Left Endpoint Rule with

- $n = 5$ ,
- $n = 25$ ,
- $n = 100$

subintervals. Provide your code and the plot for each case. Answer the question, which case provide the best estimation and why?

**Question 3:**

Complete the provided code `lab11.R` to calculate antiderivative

$$F(10) = \int_0^{10} e^{\sin(t)} dt,$$

using the Right Endpoint Rule with

- $n = 5$ ,
- $n = 25$ ,
- $n = 100$

subintervals. Provide your code and the plot for each case. Is the estimated  $F(10)$  the same as you calculated in the previous question? Why or why not?

**Question 4:**

The Midpoint Rule uses the value of the curve in the middle of each subinterval to calculate the height of the rectangle. Complete the provided code `lab11.R` to calculate antiderivative

$$F(10) = \int_0^{10} e^{\sin(t)} dt,$$

using the Midpoint Rule with

- $n = 5$ ,
- $n = 25$ ,
- $n = 100$

subintervals. Provide your code and the plot for each case.

**Note: Please adjust the size of your plots in your assignment for clean presentation.**