1. (4 points) (Problem 13) The tangent line to the graph of $f(x)$ at $x = 1$ is shown. On the tangent line, P is the point of tangency and A is another point on the line.
   (a) Find the coordinates of the points P and A.
   (b) Use the coordinates of P and A to find the slope of the tangent line.
   (c) Find $f'(1)$.
   (d) Find the instantaneous rate of change of $f(x)$ at P.

Solution:

(a) $P = (1, 3)$ and $A = (0, 3)$.

(b) The slope of the line is $\frac{3-3}{0-1} = 0$.

(c) $f'(1)$ is the slope of the tangent line to $f(x)$ at $x = 1$. So $f'(1) = 0$.

(d) The instantaneous rate of change of $f(x)$ at $x = 1$ is the same as $f'(1)$ and the slope of the tangent line, i.e. the instantaneous rate of change if 0.

2. (1 point) (Problem 42) If the total revenue function for a blender is $R(x) = 36x - 0.01x^2$ where x is the number of units sold, what is the average rate of change in revenue $R(x)$ as x increases from 10 to 20 units?

Solution:

$$\begin{aligned}\text{Average Rate of Change} &= \frac{R(20) - R(10)}{20 - 10}\\ &= \frac{716 - 359}{10}\\ &= 35.7\end{aligned}$$

So the average rate of change between $x = 10$ and $x = 20$ is \$35.70 per additional unit produced.

3. (5 points) (Problem 46) Suppose the total revenue function for a blender is $R(x) = 36x - 0.01x^2$ dollars, where x is the number of units sold.
   (a) What function gives the marginal revenue?
   (b) What is the marginal revenue when 600 units are sold and what does it mean?
   (c) What is the marginal revenue when 2000 units are sold and what does it mean?
   (d) What is the marginal revenue when 1800 units are sold and what does it mean?

Solution:

(a) To find the marginal revenue function, we need to compute the derivative function, $R'(x)$ using the limit definition of the derivative:

$$\begin{aligned}
R'(x) &= \lim_{h\to 0} \frac{R(x+h)-R(x)}{h} \\
&= \lim_{h\to 0} \frac{[36(x+h)-0.01(x+h)^2]-[36x-0.01x^2]}{h} \\
&= \lim_{h\to 0} \frac{36x+36h-0.01x^2-0.02xh-0.01h^2-36x+0.01x^2}{h} \\
&= \lim_{h\to 0} \frac{36h-0.02xh-0.01h^2}{h} \\
&= \lim_{h\to 0} 36-0.02x-0.01h \\
&= 36-0.02x
\end{aligned}$$

(b) The marginal revenue when 600 units are sold is $R'(600) = 36-0.02(600) = 24$. This means that the 601st unit produced will bring in \$24 additional revenue.

(c) The marginal revenue when 2000 units are sold is $R'(2000) = 36-0.02(2000) = -4$. This means that the revenue will decrease by \$4.00 when the 2001st unit is sold.

(d) The marginal revenue when 1800 units are sold is $R'(1800) = 36-0.02(1800) = 0$. This means that revenue is not changing when 1800 units are sold.