The following questions are designed to help you prepare for Exam 2. You should also study the relevant vocabulary, concepts, and formulas (especially for chapters 8 & 9). You will be allowed one 4 × 6 notecard in the exam. The tables of Normal Distribution values and Random Digits (Tables A & B) will be provided for you. You may wish to include the normal distribution critical ($z^*$) values on your index card.

1. You are the president of a student group on campus. You are constructing a survey to improve classroom instruction for your statistics class. To do so, you obtain a list of 1500 students who have taken a statistics class in the last two years.

(a) Describe a procedure to find a simple random sample of 500 students.

**Solution:** First, assign a 4-digit label to each student on the list, 0001 through 1500. Next, choose 500 of the students using the random digits table.

(b) Use the table of random numbers to obtain the first five students in your sample. (Start with line 132.)

**Solution:** 0418, 0765, 0705, 1320, 1337

2. Consider the experiment of tossing two fair dice.

(a) Write out the sample space $S$ of all possible outcomes.

**Solution:** See figure 10.2 in your textbook.

(b) Is this a discrete or continuous random variable?

**Solution:** Discrete.

(c) Give the probability model for this sample space.

**Solution:** Probability of any single outcome in $S$ is $\frac{1}{36}$.

(d) Let $A$ be the event that the sum of the dice is 8. Find $P(A)$.

**Solution:** $\frac{5}{36}$

(e) Find the probability that the first die is 6 and the second die is odd.
3. The number of accidents per week at a hazardous intersection varies with mean 2.2 and standard deviation 1.4. This distribution takes only whole-number values, so it is definitely not Normal. Let $\bar{x}$ be the mean number of accidents per week at the intersection during a year (52 weeks). What is the approximate distribution of $\bar{x}$?

**Solution:** Regarding the 52 weeks in a year as an SRS of 52 weeks out of all possible weeks, $\bar{x}$ is $N(2.2, \frac{1.4}{\sqrt{52}})$. (Central Limit Theorem)

4. The daily study times of students follow a Normal distribution with unknown mean $\mu$ and standard deviation of 20 minutes. A sample of 60 students is taken and the sample mean is 130 minutes.

(a) What conditions need to be met before you can do statistical inference on this data?

**Solution:**

(b) Find 80%, 85%, 90%, 95%, and 99% confidence intervals for the mean study time.

**Solution:** In order of increasing level of confidence:

$130 \pm 3.310$

$130 \pm 3.718$

$130 \pm 4.247$

$130 \pm 5.061$

$130 \pm 6.651$
(c) Based solely on your answer to part (b), what is the smallest level \( \alpha \) at which you know for certain you can reject the null hypothesis that \( \mu = 135 \) in favor of the 2-sided alternative \( \mu \neq 135 \)?

**Solution:** 95% confidence interval includes 135 but 90% does not, so we know we can reject at \( \alpha = 0.10 \) but not at \( \alpha = 0.05 \).

(d) Students claim to study 120 minutes per day. Test the hypothesis against the two-sided alternative at the 5% significance level and state your conclusion.

**Solution:** Reject the null hypothesis; the results are significant at the level \( \alpha = 0.05 \).

(e) Suppose that you want a specified margin of error of no more than 12 minutes in a 95% confidence interval. What is the smallest sample size that achieves this margin of error?

**Solution:** \( n = 11 \).

5. Suppose that you want a specified margin of error of 5.1 in your 95% confidence interval for some measurement. Assume the population standard deviation is 16. What must your sample size be to achieve this margin of error?

**Solution:** \( n = 38 \).

6. (Adapted from “Probability and Statistics” by J. L. Devore) The recommended daily dietary allowance for zinc among males older than age 50 years is 15 mg/day. The article “Nutrient Intakes and Dietary patterns of Older americans: A National Study” (*J. of Gerontology*, 1992: M145-150) reports that a sample of 115 men ages 65-74 years consumed an average of 11.3 mg/day of zinc. Assuming that zinc intakes among older men vary Normally with standard deviation \( \sigma = 6.43 \), does the data indicate (at the significance level \( \alpha = 0.01 \)) that the average daily zinc intake for men ages 65-74 falls below the recommended level?

**Solution:** Solutions to hypothesis testing problems in general should be formatted as follows:

**Step 1:** State Hypotheses

\[
H_0 : \mu = 15 \\
H_a : \mu < 15
\]
Step 2: Calculate the z-statistic: We know that $\bar{x} = 11.3, n = 115, \sigma = 6.43$.

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{11.3 - 15}{6.43 / \sqrt{115}} = \frac{-3.7}{0.5996} = -6.17$$

Step 3: Calculate the P-value:

$$P = P(Z \leq -6.17) < P(Z \leq -2.49) = 0.002$$

(Since -6.17 is not found on table A, the most precise we can be is that the P-value is less than 0.0002.)

Step 4: State conclusion: Since the P-value is smaller than $\alpha$, we reject the null hypothesis and conclude that yes, the average daily intake for men ages 65-74 most likely falls below the recommended level.

7. (Adapted from “Probability and Statistics” by J. L. Devore) An employee at Home Depot wants to determine whether it takes longer than 75 hours for a certain type of paint to dry. The drying times are Normally distributed with $\sigma = 9$ hours. In a random sample of 25 observations, it is determined that the average drying time is 72.3 hours. Is there sufficient evidence to conclude that this type of paint takes longer than 75 hours to dry?

Solution:

Step 1: State Hypotheses

$$H_0 : \mu = 75 \text{ vs. } H_a : \mu > 75$$

Step 2: Calculate z-statistic: We know that $\bar{x} = 72.3, n = 25, \sigma = 9$.

$$z = \frac{72.3 - 75}{9 / \sqrt{25}} = \frac{-2.7}{1.8} = -1.5$$

Step 3: Calculate P-value: Since we are testing the alternative hypothesis $H_a : \mu > 75$, we want to know the probability of finding a z-score as large, or larger than the
statistic we calculated in Step 2. In other words, we want to find the area under the standard normal curve to the right of our z-score, -1.5.

\[ P(Z \geq -1.5) = 1 - P(Z \leq -1.5) = 1 - .0668 = .9332 \]

**Step 4: State Conclusion:**
Since the \( P \)-value is very large (.9332), we do not reject the null hypothesis. There is very little evidence that this type of paint takes more than 75 hours to dry.

**Note:** Based on the facts that the sample mean was less than \( \mu_0 \), and we were testing the hypothesis that \( \mu \) is greater than \( \mu_0 \), we can predict that the \( P \)-value will be at least .50, and therefore we would fail to reject the null hypothesis at any significance level.

**For more examples of hypothesis testing, see Ch 14-15 examples from class. Make sure you know the conditions for applying these procedures (Normality, SRS, known \( \sigma \), unknown \( \mu \)).**

**For examples of experimental design, see exercises in Ch 8-9. Make sure you know the different types of designs (matched pairs, block, etc.), sampling techniques (SRS, stratified random sample, etc.), and potential pitfalls (response bias, sampling bias, undercoverage, etc.).**