Name: Solutions UID: _____

1. T

2. F

3. T

4. F

5. T

6. F

7. T

8. F

9. $(-23, 9)$

10. $\begin{pmatrix} 3 & 3 \\ -3 & 2 \end{pmatrix}$

11. $\sqrt{50}$

12. $(y-4) = -(x-2)$

13. $y = 4x$

14. $x^2 + y^2 = 1$

15. $x^2 + y^2 = 9$

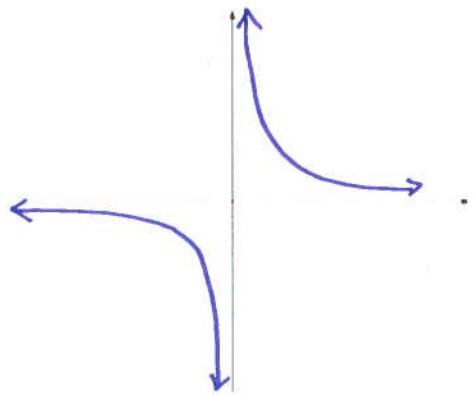
16. $(x+1)^2 + (y-2)^2 = 16$

17. $(3x)^2 + \left(\frac{1}{3}y\right)^2 = 1$

$\left(9x^2 + \frac{y^2}{9} = 1\right)$

****

18.



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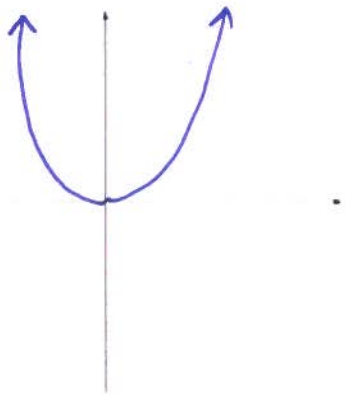

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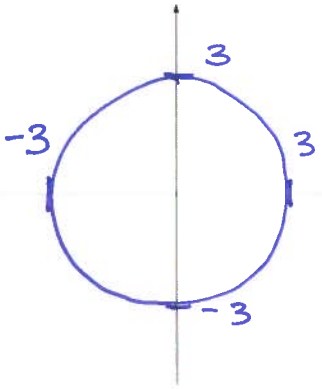

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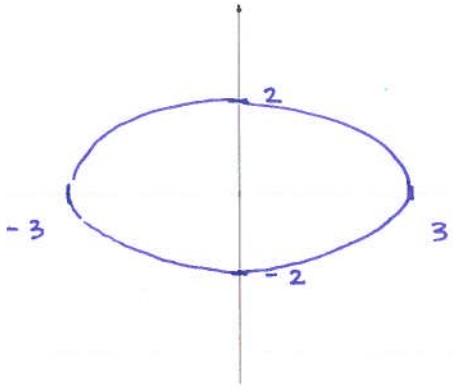

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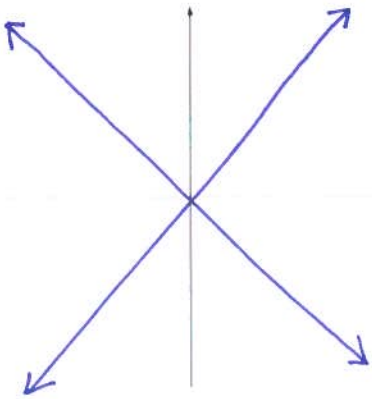

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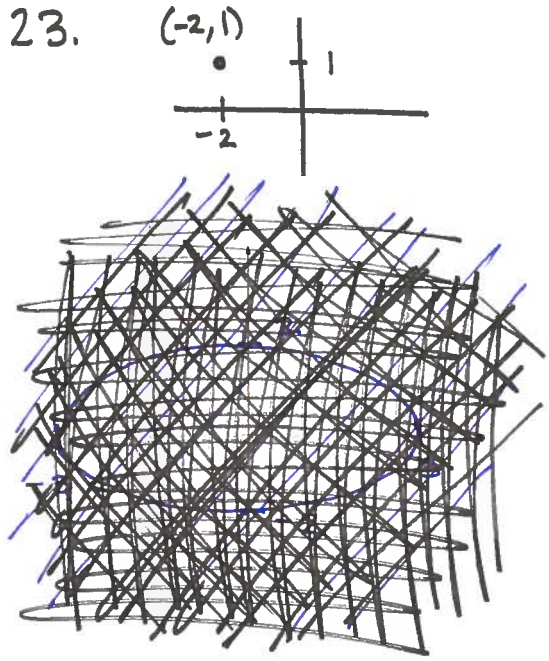

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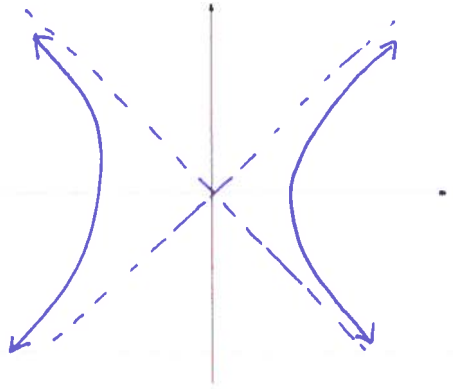

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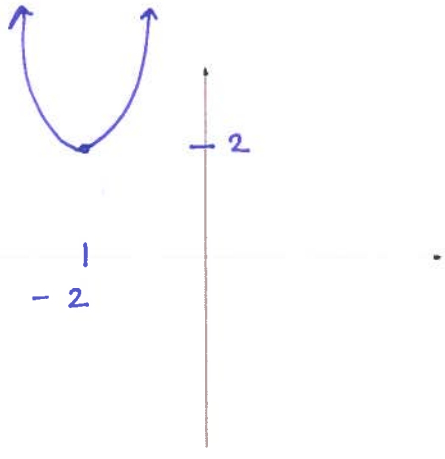


26. $\sqrt{17}$

27. 6

28. $\frac{3}{4}$

29. $\sqrt{15}/4$

30. 2

31. $\sqrt{2}$

32. $\sqrt{3}$

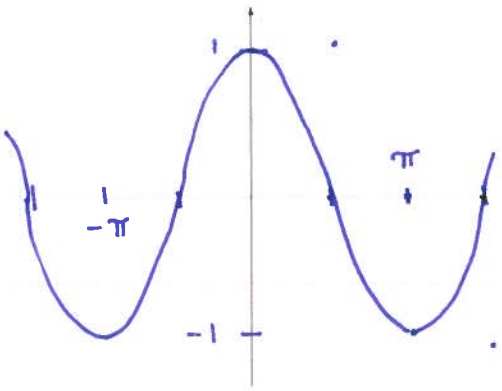
33. $\frac{\pi}{6}$

34. $\frac{\pi}{6}$

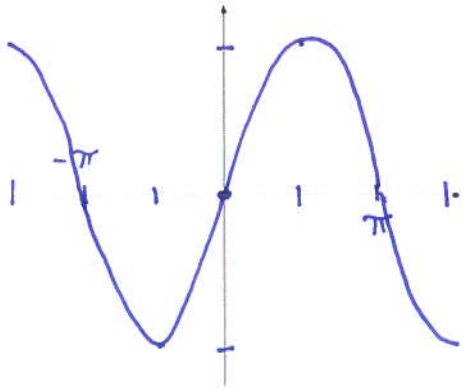
35. $-\frac{\pi}{4}$

****

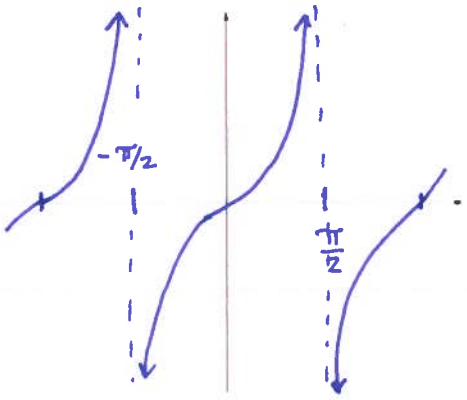
36. $\cos(x)$



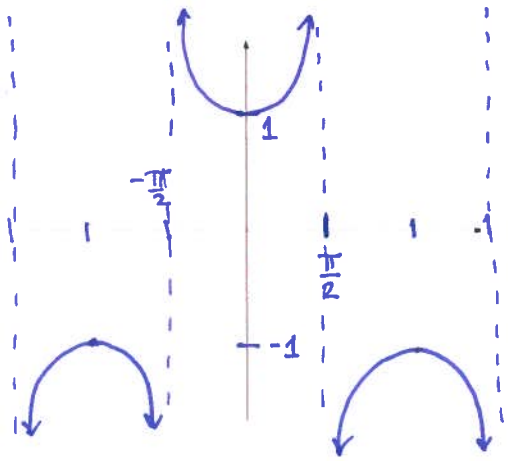
37. $\sin(x)$



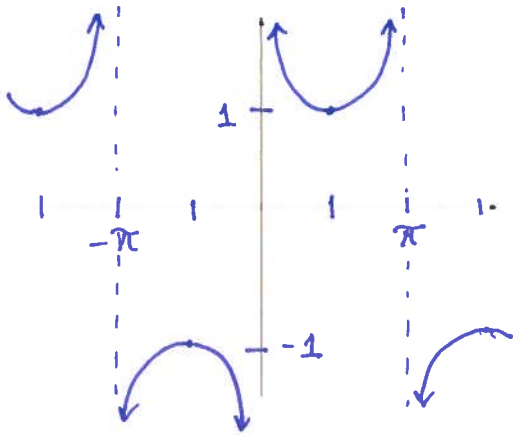
38. $\tan(x)$



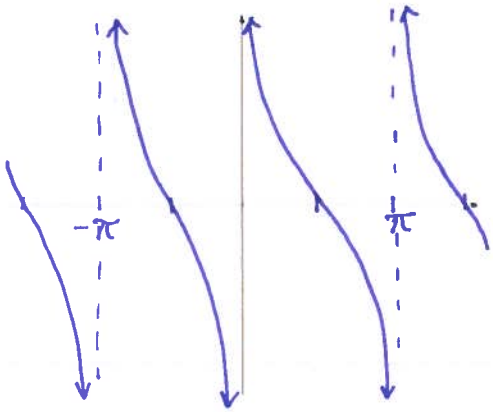
39. $\sec(x)$



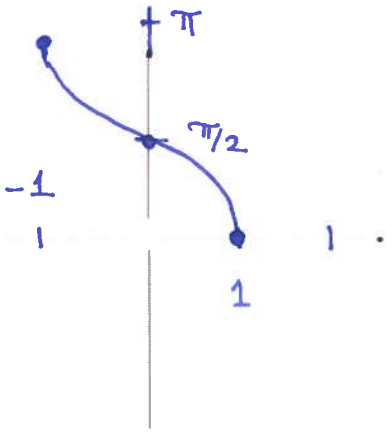
40. $\csc(x)$



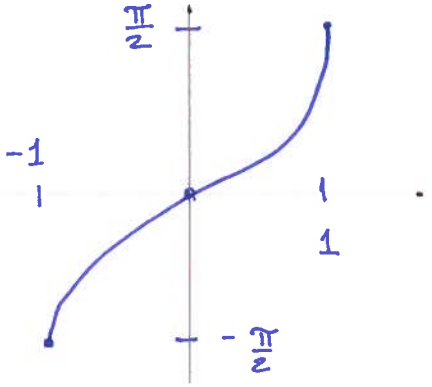
41. $\cot(x)$



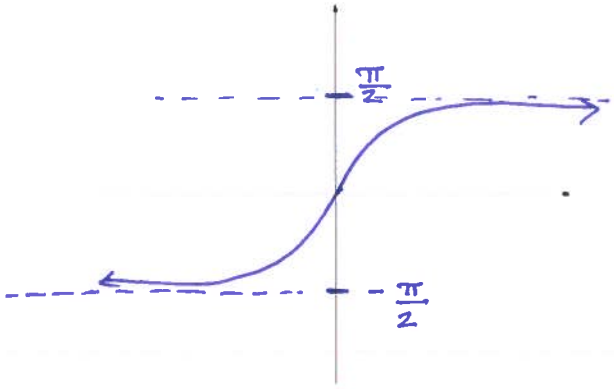
42. $\arccos(x)$



43. $\arcsin(x)$



44. $\arctan(x)$



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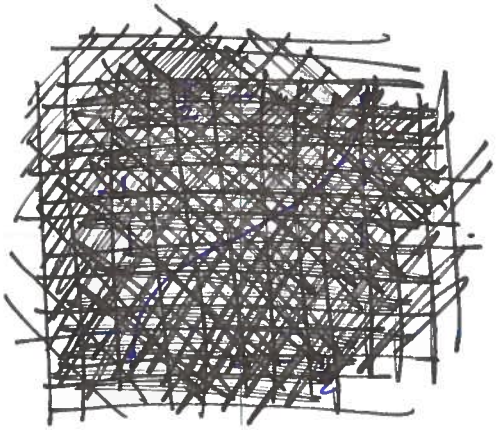


45. A

46. H

47. C

48. F

49. E

50. D

51. G

52. H

53. I

54. J

55. B

56. A

57. A

58. B

59. $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

60. $-\frac{4\sqrt{5}}{9}$

Find all solutions to the following equations. If there are no solutions, write one sentence explaining why.

61. $\log_{10}(x-2)e^x = 2\log_{10}(x-2)$

Implied domain

$x-2>0$

$x>2$

$(2,\infty)$

Z = zeros of $\log_{10}(x-2)$ in $(2,\infty)$:

$\log_{10}(x-2)=0$

$x-2=1$

$x=3$

$Z=\{3\}$

$e^x = 2$

$x = \log_e(2)$

$\log_e(2) < 1$, since $2<e$

So $\log_e(2)$ is not a solution.

$\{3\}$

62. $x^2+4=0$

$x^2 = -4$

No solutions. even powers are never negative.

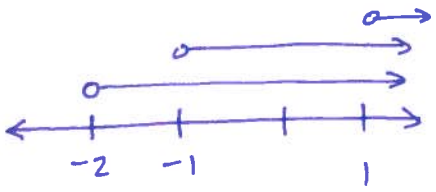
63. $\log_2(x+2) = \log_2(x-1) - \log_2(x+1)$

Implied domain:

$x+2>0$
$x>-2$

$x-1>0$
$x>1$

$x+1>0$
$x>-1$



overlap: $(1,\infty)$

$\log_2(x+2) = \log_2\left(\frac{x-1}{x+1}\right)$

$0 = \log_2\left(\frac{x-1}{x+1}\right) - \log_2(x+2)$

$0 = \log_2\left(\frac{x-1}{(x+1)(x+2)}\right)$

$1 = 2^0 = \frac{x-1}{x^2+3x+2}$

$x^2+3x+2 = x-1$ $\quad b^2-4ac = 4-12<0$

$x^2+2x+3 = 0$

No solutions because the discriminant is negative.

64. $2(e^x)^2 - 3e^x + 1 = 0$

This is a quadratic equation in e^x, and it has domain $\mathbb{R}$.
If we substitute y for e^x, we have: $2y^2 - 3y + 1 = 0$
The solutions to this equation are

$y = \frac{3+\sqrt{9-8}}{4} = 1$ and $y = \frac{3-\sqrt{9-8}}{4} = \frac{1}{2}$

Switching back from y to e^x:

$e^x = 1$
$\Downarrow$
$x = 0$

$e^x = \frac{1}{2}$
$\Downarrow$
$x = \log_e\left(\frac{1}{2}\right)$

$\left\{0, \log_e\left(\frac{1}{2}\right)\right\}$

65. $\log_3(1-x) = 2 - \log_3(x-4)$

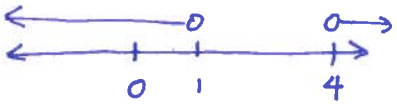
Implied domain:

$1-x>0$

$1>x$

$x-4>0$

$x>4$



Overlap = $\emptyset$

Since the implied domain is $\emptyset$, this equation has no solutions.

66. $\log_e(x-1)^2 = 9$

Implied domain:

$x-1>0$

$x>1$

$(1,\infty)$

$\log_e(x-1) = 3$ or $\log_e(x-1) = -3$

$x-1 = e^3$ $\qquad$ $x-1 = e^{-3}$

$x = e^3+1$ $\qquad$ $x = 1+\frac{1}{e^3}$

since both e^3+1 and $\frac{1}{e^3}+1$ are in the implied domain, both are solutions.

$\left\{e^3+1, \frac{1}{e^3}+1\right\}$

Answer all questions below. All questions are worth 1 point except where otherwise noted. No cell phones, calculators, or notes are allowed during the exam. If you are stuck on a problem, skip it and come back to it later. The exam will last 120 minutes.

Name: Solutions UID: _____

Write all your answers in the answer booklet provided.

True or False

Decide whether each statement is true or false. Worth $\frac{1}{2}$ point each.

1. The point $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$ lies on the unit circle. T
2. If S is the set of solutions to the equation $x^3 + y^3 = xy$, then $(-1, 1) \in S$. F
3. The planar transformation $A_{(2,3)}$ shifts up by 3 and right by 2. T
4. The matrix $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ rotates the plane clockwise by an angle of $\frac{\pi}{4}$. F
5. If θ is any real number, then $\cos(-\theta) = \cos(\theta)$. T
6. If θ is any real number, then $\sin(-\theta) = \sin(\theta)$. F
7. The equation $x^2e^x = (x+1)e^x$, with domain $\mathbb{R}$ is equivalent to the equation $x^2 = x + 1$. T
8. The equation $\log_{10}(x)^2 = 4$, with domain $(0, \infty)$, is equivalent to the equation $\log_{10}(x) = 2$. F

Linear Algebra

9. Write the resulting vector as a row vector:

$$\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 5 \end{pmatrix} =$$

$$\begin{pmatrix} -3 & -20 \\ -1 & +10 \end{pmatrix} = \begin{pmatrix} -23 \\ 9 \end{pmatrix}$$

10. Find the product:

$$\begin{pmatrix} 3 & 0 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix}$$

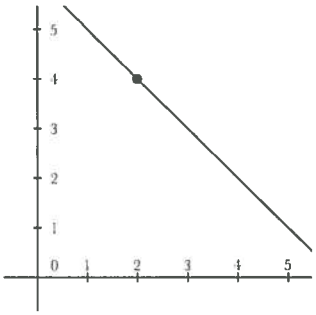
$$= \begin{pmatrix} 3\cdot 1 + 0\cdot 1 & 3\cdot 1 + 0\cdot(-4) \\ -2\cdot 1 + (-1)\cdot 1 & -2\cdot 1 + (-1)(-4) \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ -3 & 2 \end{pmatrix}$$

11. Find the norm of the vector $(1, 7)$.

$$\|(1,7)\| = \sqrt{1^2 + 7^2} = \sqrt{1+49} = \sqrt{50}$$

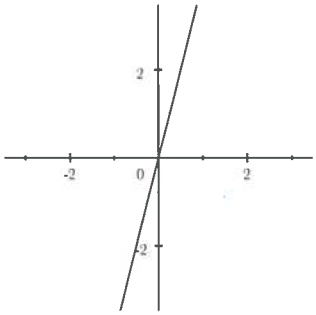
Conics and Solutions of Equations in Two Variables

12. Give the equation for a line of slope -1, through the point $(2, 4)$



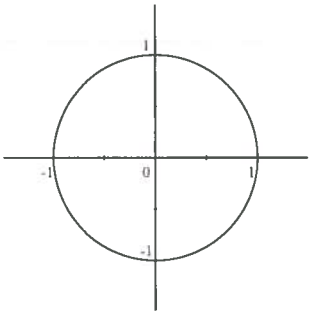
$(y-4) = -(x-2)$

13. Give the equation for the line of slope 4 through the origin.



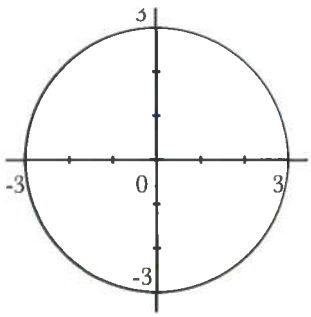
$y = 4x$

14. Give the equation for the unit circle.



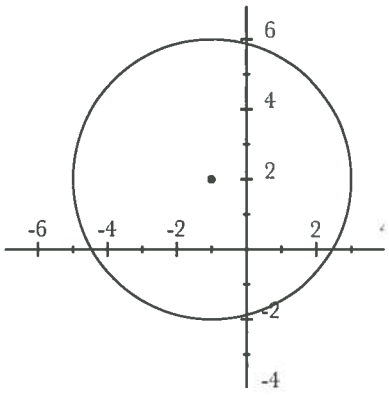
$x^2 + y^2 = 1$

15. Give the equation for the circle of radius 3 centered at the origin.



$x^2 + y^2 = 9$

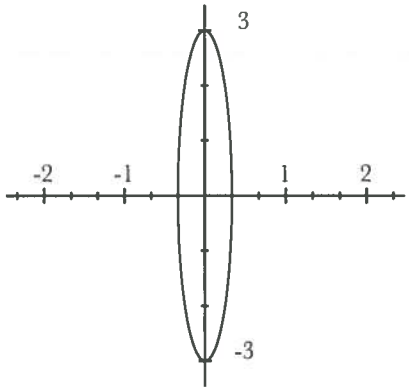
16. (2 points) Give the equation for the circle of radius 4 centered at the point $(-1, 2)$.



$A^{-1}_{(-1,2)} = A_{(1,-2)}$: $x \longmapsto x+1$, $y \longmapsto y-2$

$x^2+y^2=16 \Rightarrow (x+1)^2+(y-2)^2=16$

17. (2 points) Give the equation for the ellipse obtained by starting with the unit circle, then scaling the x-axis by $\frac{1}{3}$ and the y-axis by 3.



$\begin{pmatrix} 1/3 & 0 \\ 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & 0 \\ 0 & 1/3 \end{pmatrix}$: $x \longmapsto 3x$, $y \longmapsto \frac{1}{3}y$

$x^2+y^2=1 \Longrightarrow (3x)^2+\left(\frac{1}{3}y\right)^2=1$

18. Draw the set of solutions to the equation $xy = 1$.

19. Draw the set of solutions to the equation $y = x^2$.

20. Draw the set of solutions to the equation $x^2 + y^2 = 9$.

21. Draw the set of solutions to the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$

22. Draw the set of solutions to the equation $(x - y)(x + y) = 0$.

23. Draw the set of solutions to the equation $(x + 2)^2 + (y - 1)^2 = 0$.

24. Let H be the set of solutions to $xy = 1$ (from #18). $R_{-\frac{\pi}{4}}$ is the rotation of the plane by angle $-\frac{\pi}{4}$. Draw $R_{-\frac{\pi}{4}}(H)$.

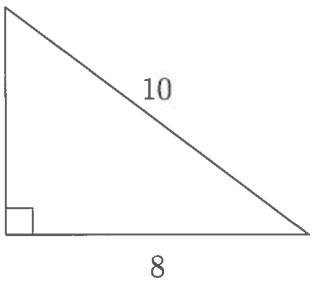
25. Let P be the set of solutions to $y = x^2$ (from #19). Draw P shifted left by 2 and up by 2.

Trigonometry

26. What is the distance from the point $(3, 3)$ to the point $(-1, 2)$?

$$\sqrt{(3-(-1))^2 + (3-2)^2} = \sqrt{4^2 + 1^2} = \sqrt{16+1} = \sqrt{17}$$

27. Find the length of the unlabeled side of the triangle below.



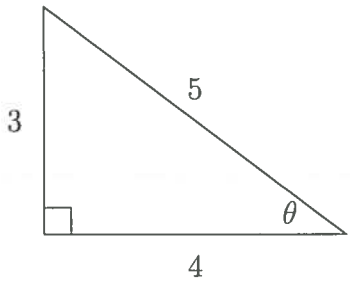
$$8^2 + x^2 = 10^2$$

$$64 + x^2 = 100$$

$$x^2 = 36$$

$$x = 6$$

28. (3 points) Find $\sin(\theta), \cos(\theta), \tan(\theta)$ for the angle θ shown below.



$$\sin(\theta) = \frac{3}{5}$$

$$\cos(\theta) = \frac{4}{5}$$

$$\tan(\theta) = \frac{3}{4}$$

29. If $\cos(\theta) = \frac{1}{4}$, and $\sin(\theta) > 0$, what is $\sin(\theta)$?

$$\cos(\theta)^2 + \sin(\theta)^2 = 1$$

$$\frac{1}{16} + \sin(\theta)^2 = 1$$

$$\sin(\theta)^2 = \frac{16}{16} - \frac{1}{16} = \frac{15}{16}$$

$$\sin(\theta) = \frac{\sqrt{15}}{4} \text{ or } -\frac{\sqrt{15}}{4} \Rightarrow \sin(\theta) = \frac{\sqrt{15}}{4}$$

30. Find $\sec(-\frac{\pi}{3})$.

$$= \frac{1}{\cos(-\frac{\pi}{3})} = \frac{1}{\frac{1}{2}} = 2$$

31. Find $\csc(\frac{\pi}{4})$.

$$= \frac{1}{\sin(\frac{\pi}{4})} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

32. Find $\tan(\frac{\pi}{3})$.

$$= \frac{\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

33. Find $\arccos(\frac{\sqrt{3}}{2})$.

$$\cos(\tfrac{\pi}{6}) = \tfrac{\sqrt{3}}{2} \Rightarrow \arccos(\tfrac{\sqrt{3}}{2}) = \tfrac{\pi}{6}$$

34. Find $\arcsin(\frac{1}{2})$.

$$\sin(\tfrac{\pi}{6}) = \tfrac{1}{2} \Rightarrow \arcsin(\tfrac{1}{2}) = \tfrac{\pi}{6}$$

35. Find $\arctan(-1)$.

$$\tan(-\tfrac{\pi}{4}) = -1 \Rightarrow \arctan(-1) = -\tfrac{\pi}{4}$$

For #36-44, graph the functions listed below.

36. $\cos(x)$
37. $\sin(x)$
38. $\tan(x)$
39. $\sec(x)$
40. $\csc(x)$
41. $\cot(x)$
42. $\arccos(x)$
43. $\arcsin(x)$
44. $\arctan(x)$

Match the functions with their graphs.

45. $\cos(x)$ A

46. $\sin(x)$ H

47. $\cos(\frac{x}{2})$ C

48. $\frac{1}{2}\cos(x)$ F

49. $2\cos(x)$ E

50. $\cos(2x)$ D

51. $\cos(x+\frac{\pi}{2})$ G

52. $\cos(x-\frac{\pi}{2})$ H

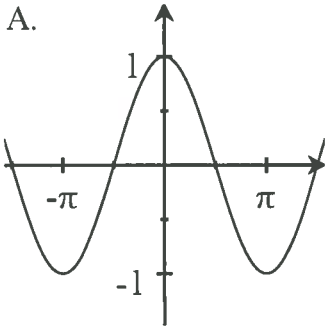
53. $\cos(x)+1$ I

54. $\cos(x)-1$ J

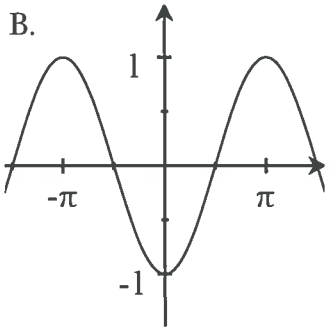
55. $-\cos(x)$ B

56. $\cos(-x)$ A

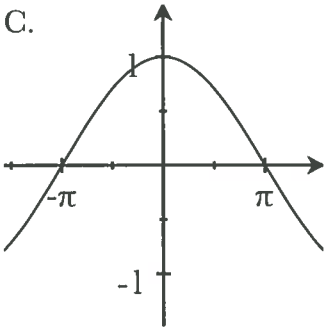
A.



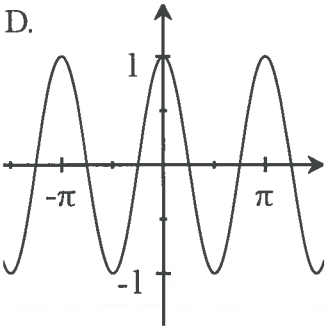
B.



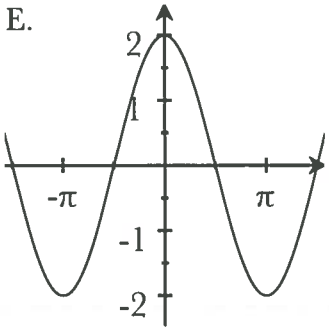
C.



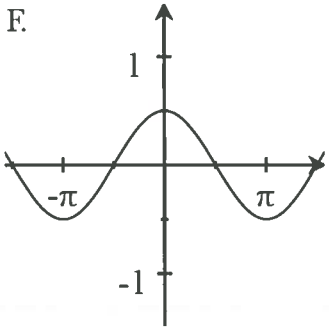
D.



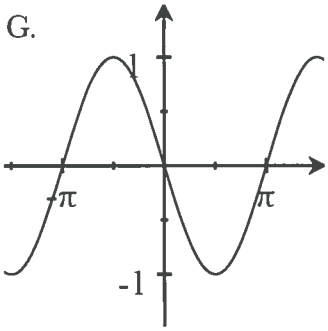
E.



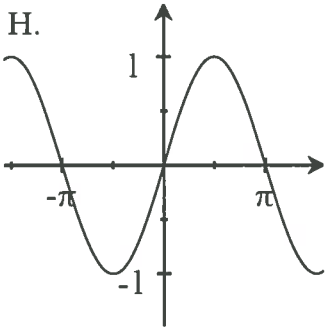
F.



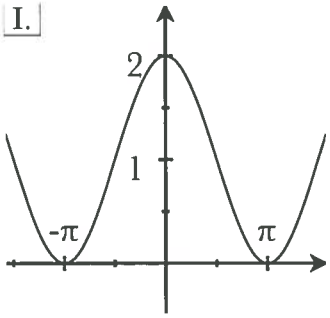
G.



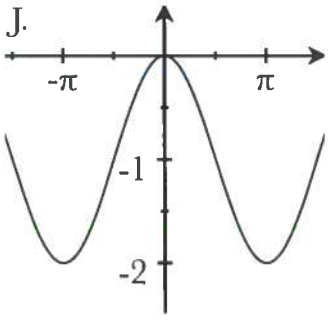
H.



I.



J.

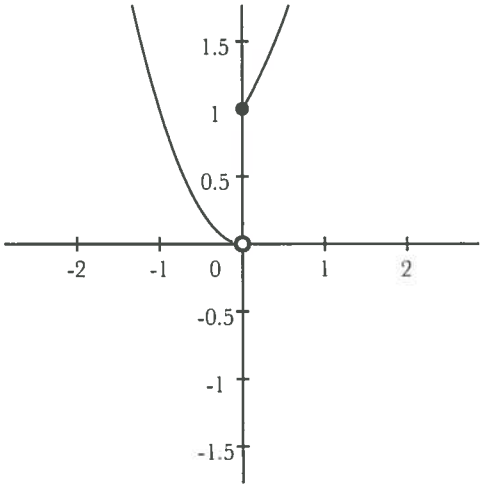


Match the functions with their graphs.

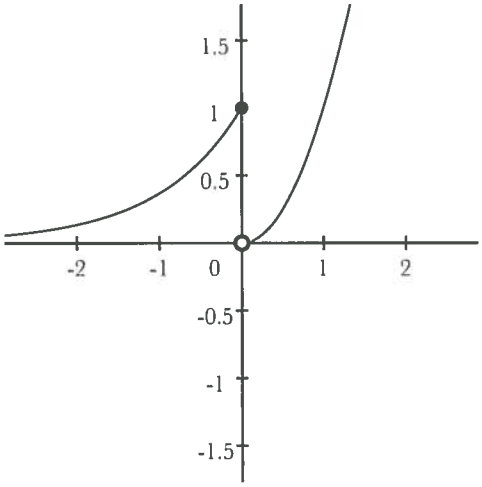
57. $f(x) = \begin{cases} e^x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$ A.

58. $g(x) = \begin{cases} e^x & \text{if } x > 0 \\ x^2 & \text{if } x \leq 0 \end{cases}$ B.

A.



B.



59. (2 points) Write the matrix that rotates the plane counter clockwise by an angle of $\frac{\pi}{3}$.

$$R_{\pi/3} = \begin{pmatrix} \cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

60. If $\sin(\theta) = \frac{2}{3}$ and $\cos(\theta) = -\frac{\sqrt{5}}{3}$, find $\sin(2\theta)$. (Hint: $\sin(2\theta) = \sin(\theta + \theta)$.)

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$

$$\sin(2\theta) = 2\sin\theta\cos\theta = 2\cdot\left(\frac{2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = -\frac{4\sqrt{5}}{9}$$

Equations in One Variable

Questions #61-66 can be found in your answer booklet.