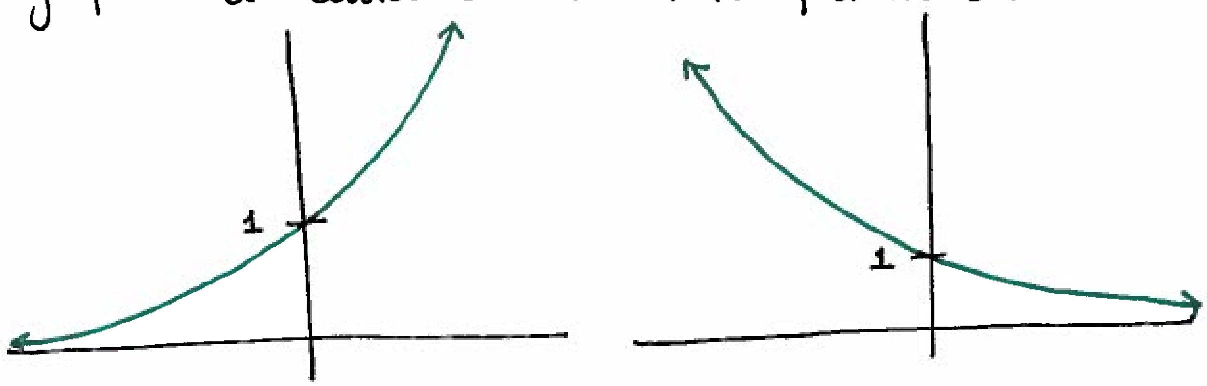
Logarithms & Exponentials

An exponential function is something like $f(x) = a^x$, where a is a real number, $a > 0$, and $a \neq 1$.

Question: Why do we want $a \neq 1$? What would the function $f(x) = 1^x$ be? Think about the values 1^x takes for different values of x.

a is called the base of the exponential function $f(x) = a^x$.

The graph of a^x looks like one of the pictures below:



$f(x) = a^x$ if $a > 1$ $\qquad$ $f(x) = a^x$ if $a < 1$

The domain of a^x is $\mathbb{R}$. (You can rember this by thinking of the graph of a^x: All possible x-values appear as the x-coordinate of some point on the graph.)

The range of a^x is $(0, \infty)$. That is, any positive real # can be written as a^x for some value of x. (You can remember this by thinking of the graph: All of the positive numbers appear as y-values of the graph; 0 and negative numbers do not.)

The inverse of $f(x) = a^x$ is $f^{-1}(x) = \log_a(x)$.

*This is where logarithms come from – they are the inverses of exponential functions.

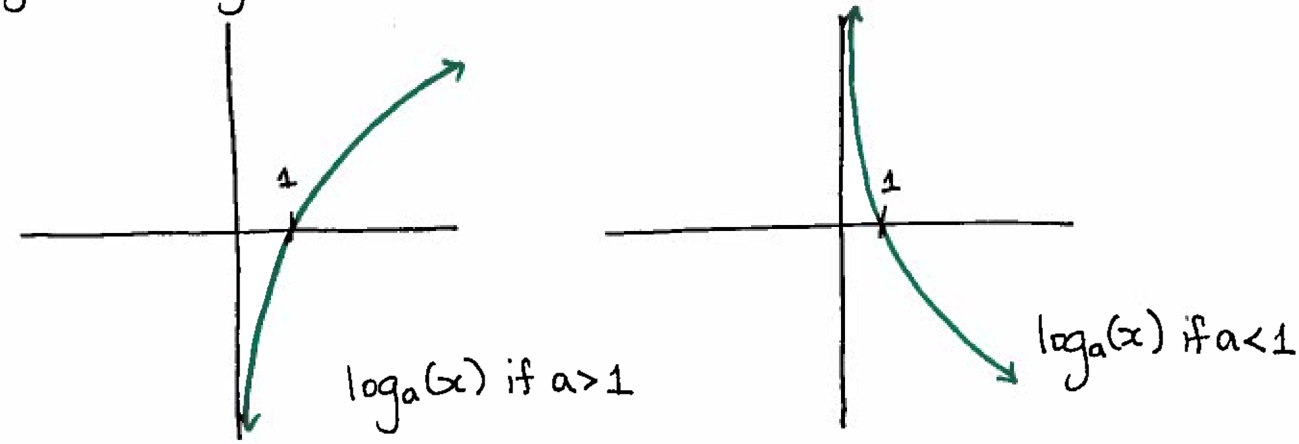
So $\quad a^{\log_a(x)} = x \quad$ & $\quad \log_a(a^x) = x$

The domain of $\log_a(x)$ is the range of its inverse: $(0,\infty)$

* This is why you can't take the logarithm of a nonpositive number (a negative number or zero): The $\log_a$ is the inverse of an exponential, so it is supposed to undo the exponential. Exponentials never give you negative numbers or zero, so logarithms can only evaluate positive numbers.

The range of $\log_a(x)$ is the domain of its inverse: $\mathbb{R}$

The graph of $\log_a(x)$ looks like one of these pictures:



Notice there are no negative x-coordinates on either graph.

Q: What does this tell you about the domain of $\log_a(x)$?

Q: What can you say about the range of $\log_a(x)$ from its graph?

A note of caution: When you are writing a logarithm, it is best to put a set of parentheses around everything you consider the "input" (like we write "$f(x)$" rather than "fx".).
On the flip side, treat the parenteses the way you would treat them for $f(x)$.

$\log_3(x^2)$	The exponent (2) happens before $\log_3()$ does. This can be rewritten as $2\log_3(x)$.
$\log_3(x)^2$	The exponent happens after $\log_3()$. This is not the same as $2\log_3(x)$.

Exercises

1) $\log_2(2^4) =$

2) $\log_2(2)^4 =$

3) $\log_{10}(100) =$

4) $\log_{10}\left(\frac{1}{1{,}000}\right) =$

5) $\log_e(e^2) =$

6) $(\log_e(e))(\log_e(e)) =$

7) $\log_{10}(2) + \log_{10}(5) =$

8) $\log_5(15) - \log_5(3) =$

9) $10^{-2} =$

10) $2^{-4} =$

1) 4 ; 2) 1 ; 3) 2 ; 4) -3 ; 5) 2 ; 6) 1 ; 7) $\log_{10}(2 \cdot 5) = 1$; 8) $\log_5\left(\frac{15}{3}\right) = 1$; 9) $\frac{1}{100}$; 10) $\frac{1}{16}$