Final Exam Review

Circle the equations with no solution:

① $e^{2x+7}=5$

② $e^{2x+7}=0$

③ $e^{2x+7}=-3$

④ $\sqrt{2x+7}=5$

⑤ $\sqrt{2x+7}=0$

⑥ $\sqrt{2x+7}=-3$

⑦ $(2x+7)^2=5$

⑧ $(2x+7)^2=0$

⑨ $(2x+7)^2=-3$

⑩ $\log_e(2x+7)=5$

⑪ $\log_e(2x+7)=0$

⑫ $\log_e(2x+7)=-3$

⑬ $\tan(x)=5$

⑭ $\tan(x)=0$

⑮ $\tan(x)=-3$

⑯ $\cos(x)=\frac{3}{2}$

⑰ $\cos(x)=0$ $\quad -1 \le \cos(x) \le 1$

⑱ $\cos(x)=-\frac{3}{2}$

⑲ $\sin(x)=\frac{1}{2}$

Use $e^x e^y = e^{x+y}$ to simplify:

① $e^x e^{-x+2} = e^{x-x+2} = e^2$

② $e^{3x-1} e^{-2x+4} = e^{3x-1-2x+4} = e^{x+3}$

③ $e^{3x+1} e^{4x-2} = e^{3x+1+4x-2} = e^{7x-1}$

Use $\log_e(x)+\log_e(y)=\log_e(xy)$ to simplify:

① $\log_e(x)+\log_e(x-2) = \log_e(x^2-2x)$

② $\log_e(x-1)+\log_e(x+1) = \log_e(x^2-1)$

③ $\log_e(x-2)+\log_e(x+3) = \log_e(x^2+x-6)$

Use the quadratic formula to solve for x:

① $x^2+x-7=0$

$b^2-4ac = 1-4(-7) = 29$

$x=\frac{-1+\sqrt{29}}{2}$ or $x=\frac{-1-\sqrt{29}}{2}$

② $x^2+2x-3=0$

$b^2-4ac = 4-(4)(-3) = 4+12 = 16$ $\qquad x=\frac{-2+4}{2}=+1$ or $x=\frac{-2-4}{2}=-3$

③ $\log_3(x)^2+2\log_3(x)-3=0$ (same coefficients as ②)

$\log_3(x)=1 \Rightarrow x=3$ or $\log_3(x)=-3 \Rightarrow x=3^{-3}=\frac{1}{27}$

Find the set of solutions

① $(2x-4)^2 = 9$ $\quad \left\{ \frac{7}{2}, \frac{1}{2} \right\}$

② $\sqrt{2x+7} = -3$ $\quad$ No solutions

③ $e^{3x+1} e^{4x-2} = 1 \Rightarrow e^{7x-1} = 1 \Rightarrow 7x-1=0 \Rightarrow x = \frac{1}{7}$

④ $(e^x)^2 + 2e^x - 3 = 0$ $\quad e^x = 1$ or $-3 \Rightarrow x = \log_e(1)$ (exponentials can't be negative so $e^x = -3$ has no solution)

⑤ $\log_e(x-2) + \log_e(x+3) = 0$

$\log_e((x-2)(x+3)) = 0 \Rightarrow x^2+x-6 = 1 \Rightarrow x^2+x-7=0$ $\quad x = \frac{-1+\sqrt{29}}{2}$ or $x = \frac{-1-\sqrt{29}}{2}$

⑥ $(x-1)(2x+3) = (x-1)(x+7)$

$\{1, 4\}$

⑦ $(2x+1)(x-4) = (2x+1)(3x)$

$\left\{ \frac{-1}{2}, -2 \right\}$

Sketch the graphs

① $\cos(x)$

② $\sin(x)$

③ $\cos\left(\frac{x}{2}\right)$

④ $\frac{1}{2}\cos(x)$

⑤ $2\cos(x)$

⑥ $\cos(2x)$

⑦ $\cos\left(x+\frac{\pi}{2}\right)$

⑧ $\cos\left(x-\frac{\pi}{2}\right)$

⑨ $\cos(x)-1$

⑩ $\cos(x)+1$

⑪ $-\cos(x)$

⑫ $\cos(-x)$

⑬ $\sin(-x)$

⑭ $-\sin(x)$

⑮ $\sin\left(x+\frac{\pi}{2}\right)$

⑯ $\sin\left(x-\frac{\pi}{2}\right)$

⑰ $\tan(x)$

⑱ $\csc(x)$

⑲ $\sec(x)$

⑳ $\cot(x)$

㉑ $\arcsin(x)$

㉒ $\arccos(x)$

㉓ $\arctan(x)$

If $\sin(\theta) = \frac{-1}{3}$ and $\cos(\theta) = \frac{2\sqrt{2}}{3}$, find $\sin(2\theta)$ and $\cos(2\theta)$.

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2\left(\frac{-1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{-4\sqrt{2}}{9}$

$\cos(2\theta) = \cos(\theta)^2 - \sin(\theta)^2 = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$

Write down the planar transformation with the given description:

① Scales the x-coordinate by 2, y-coordinate by $1/3$.

$$\begin{pmatrix} 2 & 0 \\ 0 & 1/3 \end{pmatrix}$$

② Shifts left 2, up 3.

$$A_{(-2,3)}$$

③ Flip over y-axis

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

④ Flip over the line $y=x$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

⑤ Flip over x-axis.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

⑥ Rotates the plane counterclockwise by $\frac{\pi}{2}$

$$\begin{pmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

⑦ Rotates the plane by π.

$$\begin{pmatrix} \cos(\pi) & -\sin(\pi) \\ \sin(\pi) & \cos(\pi) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

⑧ Rotates the plane by $\frac{2\pi}{3}$.

$$\begin{pmatrix} \cos(\frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}) \\ \sin(\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

⑨ Rotates the plane by $-\frac{\pi}{4}$.

$$\begin{pmatrix} \cos(-\frac{\pi}{4}) & -\sin(-\frac{\pi}{4}) \\ \sin(-\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

⑩ Shifts right 2, down 1.

$$A_{(2,-1)}$$

Draw the set of solutions.

① $xy = 1$

② $xy = -1$

③ $x^2 - y^2 = 1$

④ $y^2 - x^2 = 1$

⑤ $x^2 + y^2 = 9$

⑥ $\frac{x^2}{4} + \frac{y^2}{5} = 1$

⑦ $x^2 + y^2 = 0$

⑧ $(x-1)^2 + (y-7)^2 = 0$

⑨ $x^2 + y^2 = -1$

⑩ $y = x^2$

⑪ $x = y^2$

⑫ $x^2 - y^2 = 0$

⑬ $(x-2)(y-3) = 1$

⑭ $(x+2)^2 + (y-1)^2 = 16$

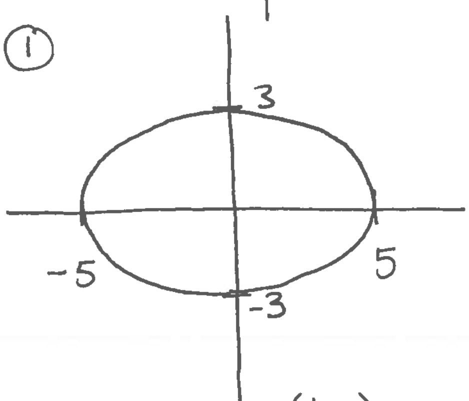
⑮ $\frac{(x-3)^2}{9} + \frac{(y+4)^2}{16} = 1$

⑯ $(y-2x+1)(y-x) = 0$

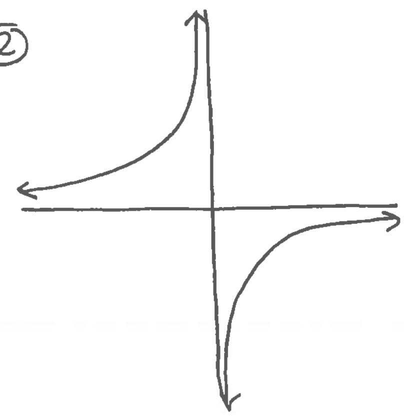
⑰ $(x-2)^2 + (y+3)^2 = -1$

Write the equations for the conics below:

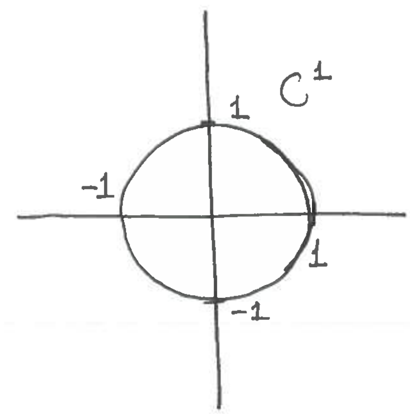
①



②



③



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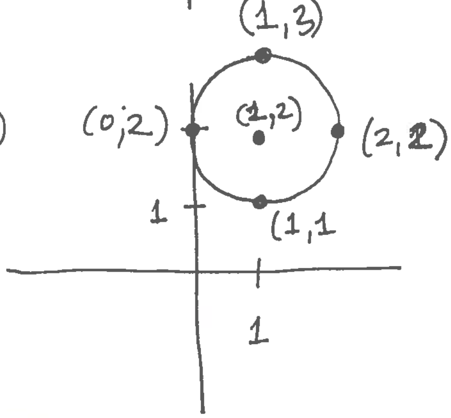
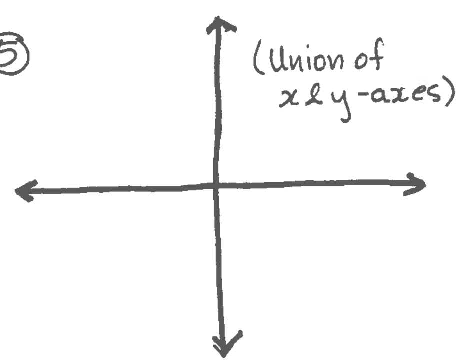
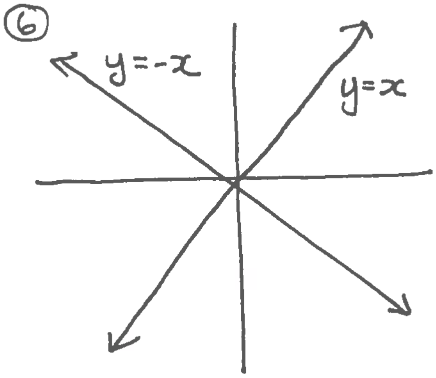


⑤



⑥



① $\frac{x^2}{25} + \frac{y^2}{9} = 1$

② $xy = -1$

③ $x^2 + y^2 = 1$

④ $(x-1)^2 + (y-2)^2 = 2^2$

⑤ $xy = 0$

$(x=0) \cup (y=0)$

⑥ $(y-x)(x+y) = 0$

$y^2 - x^2 = 0$ or

$x^2 - y^2 = 0$