Exam 2 Review

Pots, Solutions to Polynomial Equations, and Planar Transformations

Remember what POTS tells us:

> If S is the solutions to $p(x,y)=q(x,y)$
> then $T(S)$ is the solutions to $p\circ T^{-1}(x,y)=q\circ T^{-1}(x,y)$

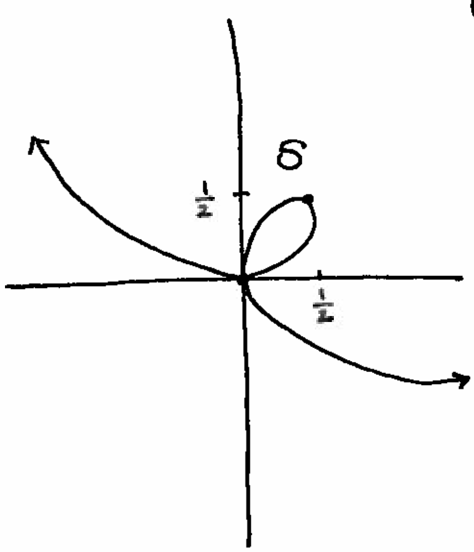
$$S \xrightarrow{T} T(S)$$
$$p(x,y)=q(x,y) \xrightarrow{T^{-1}} p\circ T^{-1}(x,y)=q\circ T^{-1}(x,y)$$

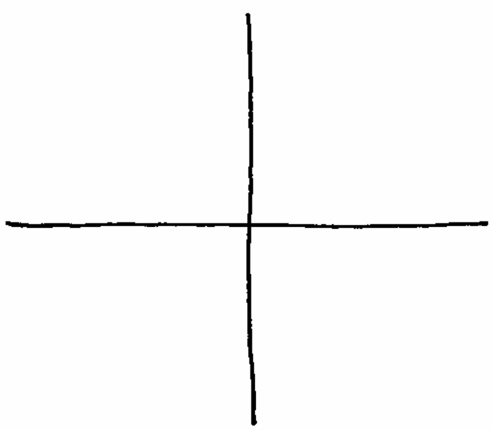
POTS is easiest to visualize when T is an addition function, flip, or a diagonal matrix, but it works for any transformation.

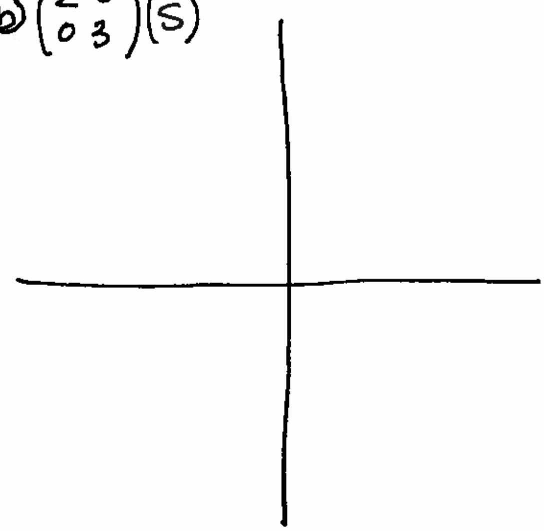
① The folium of Descartes is the set S of solutions to $x^3+y^3=xy$. Draw $T(S)$ for the transformations below.
If you're not sure how to start, first describe in words what T does to the plane.

ⓐ $A_{(2,0)}(S)$

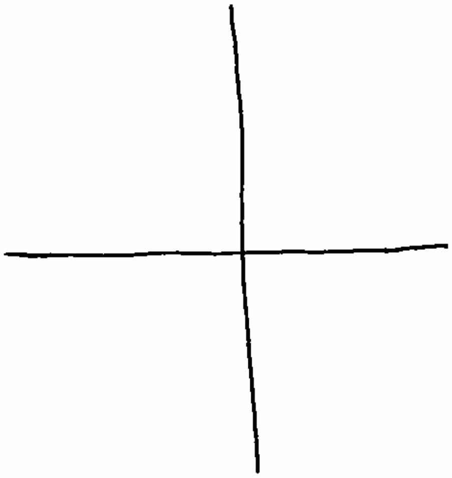
ⓑ $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}(S)$



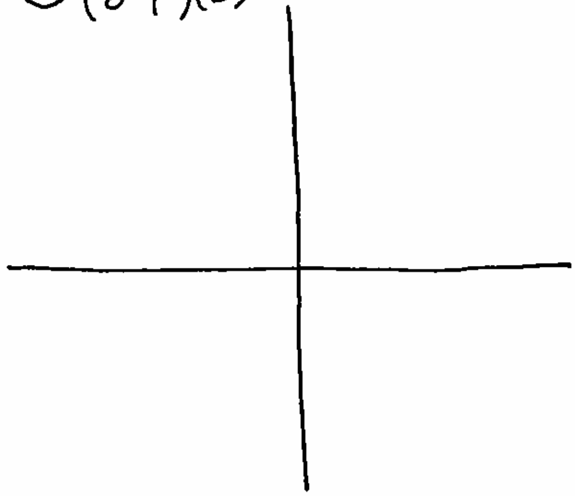




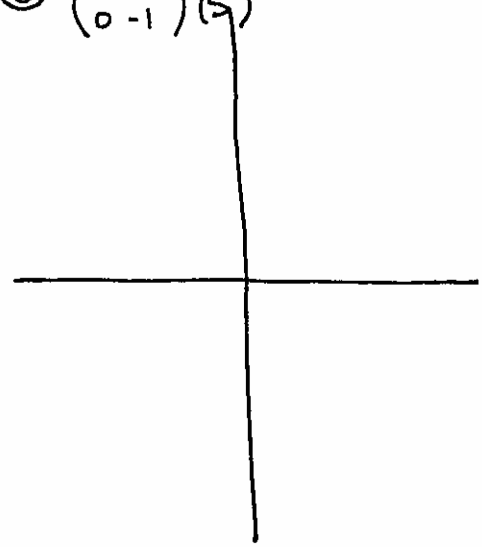
ⓒ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}(S)$



ⓓ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}(S)$



ⓔ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(S)$



② Let's see how POTS applies in these 5 situations:

To find an equation for...	I should compose the equation...	with the transformation...
$A_{(2,0)}(S)$		
$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}(S)$		
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}(S)$		
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}(S)$		
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(S)$		

③ Find the inverses below:

ⓐ $A_{(2,0)}^{-1} =$

ⓑ $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}^{-1} =$

ⓒ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} =$

ⓓ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} =$

ⓔ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} =$

④ For each transformation, find $T(x,y)$ & determine what T does to x & y.

ⓐ $A_{(-2,0)}(x,y) =$

$x \longmapsto$

$y \longmapsto$

ⓑ $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$

$x \longmapsto$

$y \longmapsto$

ⓒ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$

$x \longmapsto$

$y \longmapsto$

ⓓ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$

$x \longmapsto$

$y \longmapsto$

ⓔ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$

$x \longmapsto$

$y \longmapsto$

⑤ Use your answers to ②-④ to find an equation for $T(S)$.

ⓐ Find an equation for $A_{(2,0)}(S)$

ⓑ Find an equation for $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}(S)$

ⓒ Find an equation for $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}(S)$

ⓓ Find an equation for $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}(S)$

ⓔ Find an equation for $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(S)$.

* Remember: On page 1, the equation for S is $x^3+y^3=xy$.