Name: _____Key_____ UID: _____

1. F

2. G

3. A

4. D

5. E

6. J

7. K

8. K

9. C

10. B

11. I

12. H

13. $y = 4x$

14. $y - 5 = -7(x+2)$

15. $-\dfrac{4}{7}$

16. $y - 6 = -\dfrac{4}{7}(x+3)$
    OR $y - 2 = -\dfrac{4}{7}(x-4)$

17. $\sqrt{85}$
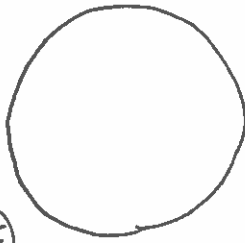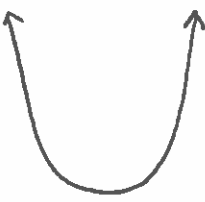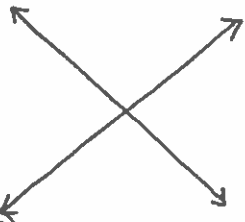
18. ~~~~ 5

19. $\dfrac{1}{2}$

20. $\dfrac{5\pi}{12}$

21. $(0,1)$

22. $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

23. $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$

24. $\left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right)$

25. $(x-3)^3 + (y+1)^3 = (x-3)(y+1)$

26. $(2x)^3 + \left(\dfrac{1}{4}y\right)^3 = (2x)\left(\dfrac{1}{4}y\right)$

Equations in One Variable
Find the solutions of the given equations and show your work. If an equation has no solution, explain why. You do not need to simplify your answers.

27. $log_3(x - 7) = 4$

$$x - 7 = 3^4$$

$$x = 7 + 3^4$$

28. $(2x - 5) = 16$

$$2x = 11$$

$$x = \frac{11}{2}$$

29. $\sqrt{3x^2 - 2} = -3$

No solution.

$\sqrt{}$ cannot be negative

30. $\frac{\frac{x}{x+1} + x}{x-2} = 1$

$$\frac{x}{x+1} + x = x - 2$$

$$\frac{x}{x+1} = -2$$

$$x = -2(x+1)$$

$$x = -2x - 2$$

$$3x = -2$$

$$x = \frac{-2}{3}$$

> Answer all questions below. All questions are worth 1 point except where otherwise noted. No cell phones, calculators, or notes are allowed during the exam. If you are stuck on a problem, skip it and come back to it later.
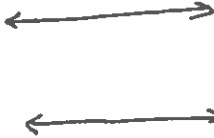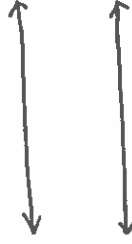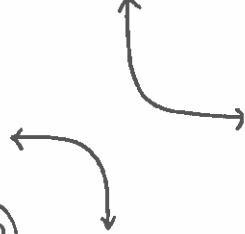
Name: ____Key_____ UID: _____

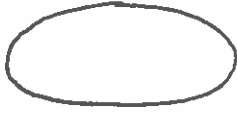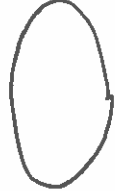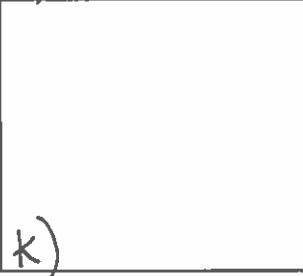Write your answers to #1-24 on the answer sheet provided.

Conics

For #1-12 match the numbered quadratic equations in two variables with their lettered sets of solutions. Worth $\frac{1}{2}$ point each.

1. $y = x^2$ **F**

2. $x^2 - y^2 = 0$ [1]   **G**

3. $x^2 = 0$ **A**

4. $xy = 1$ **D**

5. $x^2 + y^2 = 1$ **E**

6. $x^2 + y^2 = 0$ **J**

7. $x^2 + y^2 = -1$ **K**

8. $x^2 = -1$ **K**

9. $x^2 = 1$ **C**

10. $y^2 = 1$ **B**

11. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **I**

12. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ **H**

---
[1]Hint: $x^2 - y^2 = (x + y)(x - y)$

Lines

13. Give an equation for a line in the plane that has slope 4 and passes through the point $(0,0)$.

$$y = 4x$$

14. Give an equation for the line in the plane that has slope -7 and passes through the point $(-2,5)$.

$$(y-5) = -7(x+2)$$

15. Give the slope of the line that passes through the points $(-3,6)$ and $(4,2)$.

$$\frac{6-2}{-3-4} = \frac{4}{-7} = -\frac{4}{7}$$

16. Give an equation for the line that passes through the points $(-3,6)$ and $(4,2)$.
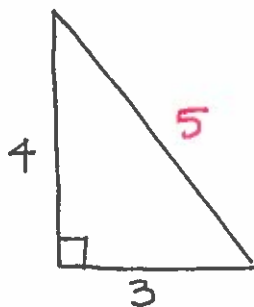
$$y - 6 = \frac{-4}{7}(x+3)$$

or

$$y - 2 = \frac{-4}{7}(x-4)$$

Trigonometry

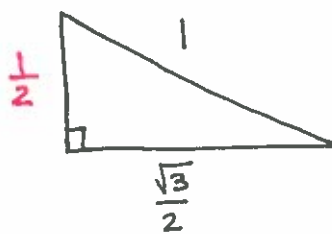17. What is the distance between the points $(4, -1)$ and $(-3, 5)$?

$$\sqrt{(4-(-3))^2 + (-1-5)^2}$$

$$= \sqrt{7^2 + 6^2}$$

$$= \sqrt{49+36} = \sqrt{85}$$

18. Find the length of the unlabeled side of the triangle below.

$$3^2 + 4^2 = x^2$$
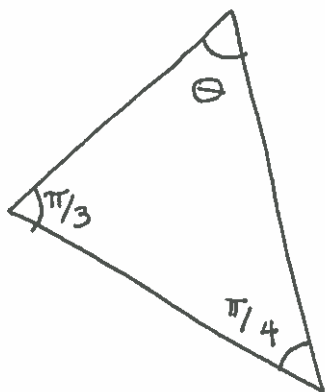$$9 + 16 = x^2$$
$$25 = x^2$$
$$x = 5$$

19. Find the length of the unlabeled side of the triangle below.

$$\left(\frac{\sqrt{3}}{2}\right)^2 + x^2 = 1^2$$

$$\frac{3}{4} + x^2 = 1$$

$$x^2 = \frac{1}{4} \implies x = \frac{1}{2}$$

20. Find the angle θ labelled below.

$$\frac{\pi}{3} + \frac{\pi}{4} + \theta = \pi$$

$$\frac{\pi}{4} + \theta = \frac{2\pi}{3}$$

$$\theta = \frac{2\pi}{3} - \frac{\pi}{4} = \frac{8\pi}{12} - \frac{3\pi}{12}$$

$$= \frac{5\pi}{12}$$

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For #21-24, find $wind(\theta)$ for the given angles. You may use the pictures of the unit circle that are attached to your answer sheet.

21. Find $wind(\frac{\pi}{2})$

$$\left(0, 1\right)$$

22. Find $wind(\frac{2\pi}{3})$

$$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

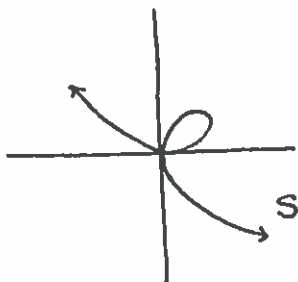23. Find $wind(-\frac{\pi}{6})$

$$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$$
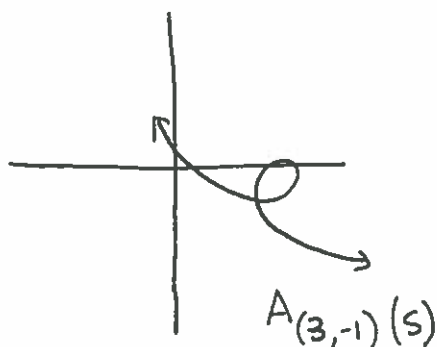
24. Find $wind(\frac{7\pi}{4})$

$$\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

Transformations of Solutions of Equations in Two Variables

The remaining questions are worth 2 points. The Folium of Descartes is the set of solutions, S, of the polynomial equation $x^3 + y^3 = xy$.
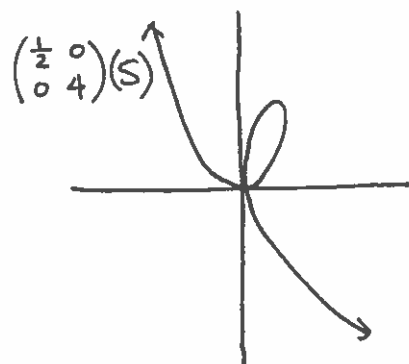

25. Give an equation for $A_{(3,-1)}(S)$, the Folium of Descartes shifted right 3 and down 1.

$$A_{(3,-1)}^{-1} = A_{(-3,1)} : \quad x \longmapsto x-3$$
$$y \longmapsto y+1$$

$$(x-3)^3 + (y+1)^3 = (x-3)(y+1)$$

$A_{(3,-1)}(S)$

26. Let $D = \begin{pmatrix} 1/2 & 0 \\ 0 & 4 \end{pmatrix}$. Give an equation for $D(S)$, the Folium of Descartes scaled by $\frac{1}{2}$ in the x-coordinate and 4 in the y-coordinate.

$$D^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1/4 \end{pmatrix} : \quad x \longmapsto 2x$$
$$y \longmapsto \tfrac{1}{4}y$$

$$(2x)^3 + \left(\tfrac{1}{4}y\right)^3 = (2x)\left(\tfrac{1}{4}y\right)$$

$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 4 \end{pmatrix}(S)$

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