Name: Key UID: _____

1. A
2. H
3. G
4. C
5. A $(-\frac{1}{2}, 1)$
6. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
7. $\begin{pmatrix} \frac{3}{2} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$
8. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
9. $(5,5)$
10. $(0,7)$
11. $(2\pi, 3)$
12. $(6,7)$
13. $(7,11)$
14. $(7,-1)$
15. $(-4,-3)$
16. $(13,17)$
17. $\begin{pmatrix} 13 & -6 \\ -7 & 4 \end{pmatrix}$
18. $27/3 = 9$
19. $\begin{pmatrix} \frac{1}{11} & \frac{-4}{11} \\ \frac{2}{11} & \frac{3}{11} \end{pmatrix}$
20. $\mathbb{R}$
21. $(0,\infty) - \{6\}$
22. $[5,\infty)$
23. $\mathbb{R} - \{0\}$
24. $[0,\infty)$

Equations in One Variable
Find the implied domain of the following equations.

20. $\log_{10}(x^2+2) = 1 - x^2$

$x^2+2 > 0$ for any x

$\Rightarrow \mathbb{R}$

21. $x^2 + 4x + \sqrt{x} = \frac{x}{x-6}$

$x > 0, \quad x \neq 6$

$(0,\infty) - \{6\}$ or $\{x \in \mathbb{R} \mid x > 0, x \neq 6\}$

22. $\sqrt{15x-3} = x^2 + 2$

$15x - 3 \geq 0$

$15x \geq 3$

$x \geq 5$

$[5,\infty)$

23. $\log_e(x^2) + 5 = x^5 + 4x + 1$

$x^2 > 0$ as long as $x \neq 0$.

$\mathbb{R} - \{0\}$

24. $e^{\sqrt{x}} = 0$

$x \geq 0$

$[0,\infty)$

The remaining questions are worth 2 points. Solve the equations in the space provided below each question.

25. $e^{2x} + 2e^x - 3 = 0$

If $(e^x)^2 + 2e^x - 3 = 0$, then $\left(b^2 - 4ac = 4 - 4(-3) = 16\right)$

$e^x = \frac{-2+\sqrt{16}}{2}$ or $e^x = \frac{-2-\sqrt{16}}{2}$

$e^x = \frac{-2+4}{2}$ → OR $e^x = \frac{-6}{2}$

$e^x = \frac{2}{2}$

$e^x = 1$

$x = 0$

which has no solutions because e^x is never negative.

So $x = 0$ is the only solution

26. $(x^2 + 2x + 1)^2 = 4$

$x^2+2x+1 = 2$ OR $x^2+2x+1 = -2$

$x^2+2x-1 = 0$

$x^2+2x+3 = 0$

$\Rightarrow b^2-4ac = 4-4(3)$

$= -8$

So $x^2+2x+3 = 0$ has no solutions.

$x = \frac{-2+\sqrt{4+4}}{2}$, $x = \frac{-2-\sqrt{4+4}}{2}$

$x = \frac{-2+\sqrt{8}}{2}$ $x = \frac{-2-\sqrt{8}}{2}$

So $x = \frac{-2+\sqrt{8}}{2}$ or $x = \frac{-2-\sqrt{8}}{2}$

27. $\log_3(1-x^2)^2 = 1$

$\log_3(1-x^2) = 1$ OR $\log_3(1-x^2) = -1$

$1-x^2 = 3^1$ $\quad$ $1-x^2 = 3^{-1}$

$1-x^2 = \frac{1}{3}$

$x^2 = -2$ $\quad$ $x^2 = \frac{2}{3}$

So $x = \sqrt{\frac{2}{3}}$ or $x = -\sqrt{\frac{2}{3}}$

28. $e^{x^2-5} = -3$

No solutions because e^{x^2-5} cannot be negative.

Answer all questions below. All questions are worth 1 point except where otherwise noted. No cell phones, calculators, or notes are allowed during the exam. If you are stuck on a problem, skip it and come back to it later.

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Write your answers to #1-24 on the answer sheet provided.

Planar Transformations

For #1-4 match each planar transformation with its geometric interpretation.

1. $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{pmatrix}$ A.
2. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ H.
3. $A_{(\frac{1}{2},2)}$ G
4. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ C.

A.) Scale x-coordinate by $\frac{1}{2}$, y-coordinate by 2.
B.) Scale x-coordinate by 2, y-coordinate by $\frac{1}{2}$.
C.) Flip over x-axis.
D.) Flip over y-axis.
E.) Flip over $y = x$ line.
F.) Moves points right 2, up $\frac{1}{2}$.
G.) Moves points right $\frac{1}{2}$, up 2.
H.) Does nothing.

For #5-8, give the inverse of the planar transformation.

5. $A_{(\frac{1}{2},-1)}$ $A_{(-\frac{1}{2},1)}$
6. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
7. $\begin{pmatrix} \frac{2}{3} & 0 \\ 0 & 4 \end{pmatrix}$ $\begin{pmatrix} \frac{3}{2} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$
8. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Matrices and Vectors

For #9-16, find the resulting vector and write it as a row vector.

9. $(6,7)+(-1,-2)$

$= (5,5)$

10. $\begin{pmatrix} -1 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -1+1 \\ -2-5 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

11. $\pi(2, \frac{3}{\pi})$

$= (2\pi, 3)$

12. $A_{(0,0)}(6,7)$

$= (6,7)$

13. $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2+9 \\ -1+12 \end{pmatrix} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$

14. $\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 1+6 \\ -1+0 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

15. $\begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

16. $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -5+18 \\ -7+24 \end{pmatrix} = \begin{pmatrix} 13 \\ 17 \end{pmatrix}$

17. Find the product: $\begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 4 & -3 \end{pmatrix}$

$$= \begin{pmatrix} -3+16 & 6-12 \\ 1-8 & -2+6 \end{pmatrix} = \begin{pmatrix} 13 & -6 \\ -7 & 4 \end{pmatrix}$$

18. Compute the determinant: $\det\begin{pmatrix} 2 & -5 \\ \frac{5}{3} & \frac{1}{3} \end{pmatrix}$

$$= 2\cdot\frac{1}{3} - (-5)\left(\frac{5}{3}\right)$$

$$= \frac{2}{3} + \frac{25}{3} = \frac{27}{3} = 9$$

19. (2 points) Find the inverse of $\begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}$.

$$\det\begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix} = 3-(4)(-2) = 3+8 = 11$$

$$\begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}^{-1} = \frac{1}{11}\begin{pmatrix} 1 & -4 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{-4}{11} \\ \frac{2}{11} & \frac{3}{11} \end{pmatrix}$$