

Name: Key UID: _____

1. A 13. (3, 5)

2. E 14. (-3, 2)

3. G 15. (7, 11)

4. D 16. 2

5. A (-2, -3)

6. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 17. $\begin{pmatrix} 6 & -12 \\ 10 & -20 \end{pmatrix}$ (2 pts)

7. $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ 18. 1

8. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ 19. $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}$ (2 pts)

9. (3, 6) 20. \mathbb{R}

10. (3, -1) 21. $[0, \infty)$

11. (-10, 15) 22. $(-\infty, 10)$

12. (3, -1) 23. $\mathbb{R} - \{0\}$

24. \mathbb{R}

The remaining questions are worth 2 points. Solve the equations in the space provided below each question.

25. $2 \log_9(x)^2 + \log_9(x) - 1 = 0$ (Write your answers to this problem as rational numbers in standard form.)

$$\log_9(x) = \frac{-1 + \sqrt{1 + 4(2)(1)}}{4} = \frac{-1 + 3}{4} = \frac{1}{2} \quad \text{OR} \quad \log_9(x) = -1$$

↓

$$x = 9^{1/2} = 3$$

$$x = 9^{-1} = \frac{1}{9}$$

$$\boxed{x = 3, x = \frac{1}{9}}$$

26. $e^{x^2-100} + 1 = 0$

$$e^{x^2-100} = -1$$

No solutions because exponentials can't be negative.

$$27. \sqrt{5x^2 + 2x - 1} = 2$$

$$5x^2 + 2x - 1 = 4$$

$$5x^2 + 2x - 5 = 0$$

$$b^2 - 4ac = 4 - 4(5)(-5) = 104$$

$$x = \frac{-2 + \sqrt{104}}{10}$$

$$x = \frac{-2 - \sqrt{104}}{10}$$

$$28. x \log_e(3x - 2) = x \text{ where } x > \frac{2}{3}.$$

$$\log_e(3x - 2) = 1$$

$$3x - 2 = e^1$$

$$3x = e + 2$$

$$x = \frac{e+2}{3}$$

$\frac{e+2}{3} > \frac{2}{3}$, so $\frac{e+2}{3}$ is in the domain.

Answer all questions below. All questions are worth 1 point except where otherwise noted. No cell phones, calculators, or notes are allowed during the exam. If you are stuck on a problem, skip it and come back to it later.

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Write your answers to #1-24 on the answer sheet provided.

Planar Transformations

For #1-4 match each planar transformation with its geometric interpretation.

- | | |
|--|---|
| 1. $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$ A | A.) Scale x -coordinate by 4, y -coordinate by 2. |
| 2. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ E | B.) Scale x -coordinate by 2, y -coordinate by 4. |
| 3. $A_{(4,2)}$ G | C.) Flip over x -axis. |
| 4. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ D | D.) Flip over y -axis. |
| | E.) Flip over $y = x$ line. |
| | F.) Moves points right 2, up 4. |
| | G.) Moves points right 4, up 2. |
| | H.) Does nothing. |

For #5-8, give the inverse of the planar transformation.

5. $A_{(2,3)}$ $A_{(-2, -3)}$
6. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
7. $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$
8. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Matrices and Vectors

For #9-15, find the resulting vector and write it as a row vector.

9. $(6, 2) + (-3, 4)$

$$(3, 6)$$

10. $\begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

11. $5(-2, 3)$

$$\begin{pmatrix} -10 \\ 15 \end{pmatrix}$$

12. $A_{(1,3)}(2, -4) = (3, -1)$

13. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

14. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

15. $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2+9 \\ -4+15 \end{pmatrix} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$

16. Compute $p_X(2, -4)$

2

17. Find the product: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 4 & -8 \end{pmatrix}$

$$\begin{pmatrix} -2+8 & 4-16 \\ -6+16 & 12-32 \end{pmatrix} = \begin{pmatrix} 6 & -12 \\ 10 & -20 \end{pmatrix}$$

18. Compute the determinant: $\det \begin{pmatrix} 3 & -1 \\ 2 & -\frac{1}{3} \end{pmatrix}$

$$3 \cdot \left(-\frac{1}{3}\right) - (2)(-1)$$

$$= -1 + 2 = 1$$

19. (2 points) Find the inverse of $\begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$.

$$\det = 0 - (-2) = 2$$

$$\frac{1}{2} \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}$$

Equations in One Variable

Find the implied domain of the following equations.

20. $x^2 - 3x + 3 = 0$

\mathbb{R}

21. $\sqrt{x} + 5 = \frac{1}{x+17}$

$[0, \infty)$

$$x \geq 0$$

$$x \neq -17$$

22. $e^{x^2-5x+2} = \log_{10}(10-x)$

$$10-x > 0$$

$$10 > x$$

$(-\infty, 10)$

23. $\log_e(x^2) + 5 = x^5 + 4x + 1$

$$x^2 > 0 \Rightarrow x \neq 0$$

$\mathbb{R} - \{0\}$

24. $e^{2x+5} = -10$

\mathbb{R}