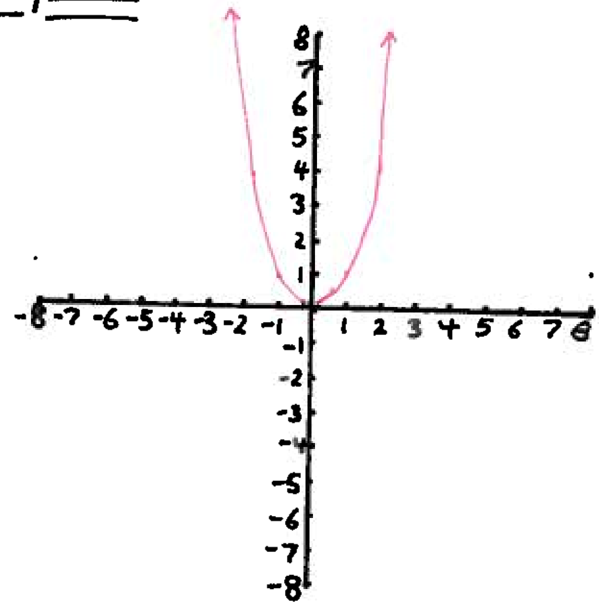
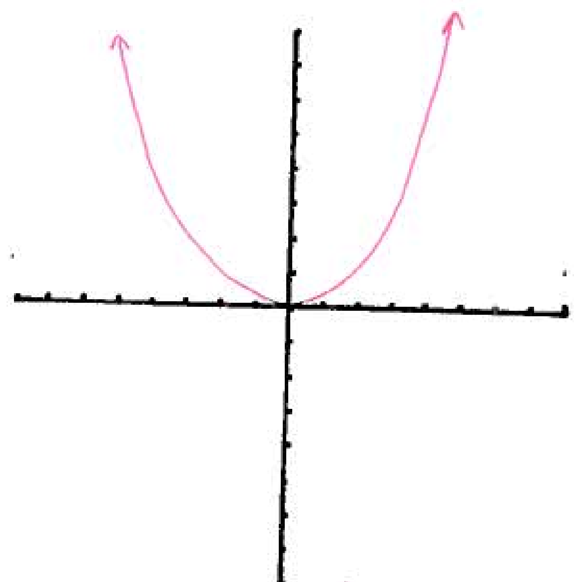
Examples of graphs

1.) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

x	$f(x)$	$(x, f(x))$
-2	$f(-2) = (-2)^2 = 4$	$(-2, 4)$
-1	$(-1)^2 = 1$	$(-1, 1)$
0	$0^2 = 0$	$(0, 0)$
1	$1^2 = 1$	$(1, 1)$
2	$2^2 = 4$	$(2, 4)$

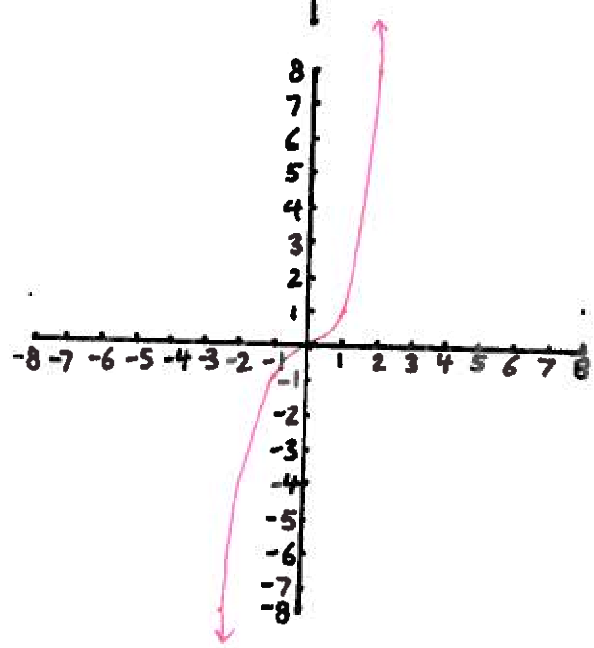


2.) Graph of
$g: \mathbb{R} \to \mathbb{R}$, $g(x) = x^n$ where
$n \in \mathbb{N}$ is even looks similar
to graph of $f(x) = x^2$

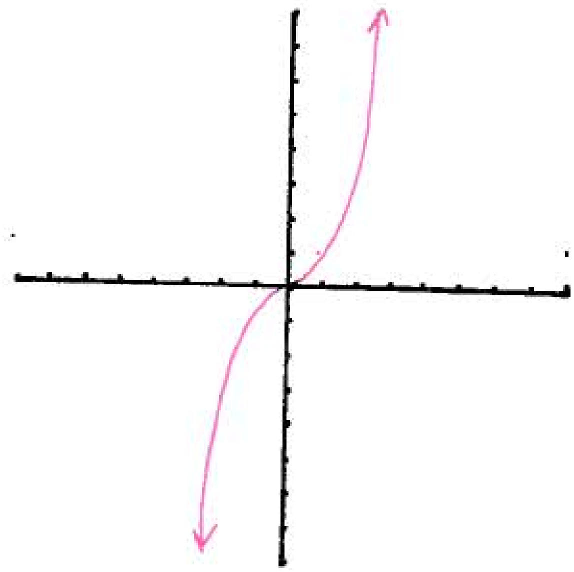


3.) $h: \mathbb{R} \to \mathbb{R}$, $h(x) = x^3$

x	$h(x)$	$(x, h(x))$
-2	$(-2)^3 = -8$	$(-2, -8)$
-1	$(-1)^3 = -1$	$(-1, -1)$
0	$0^3 = 0$	$(0, 0)$
1	$1^3 = 1$	$(1, 1)$
2	$2^3 = 8$	$(2, 8)$

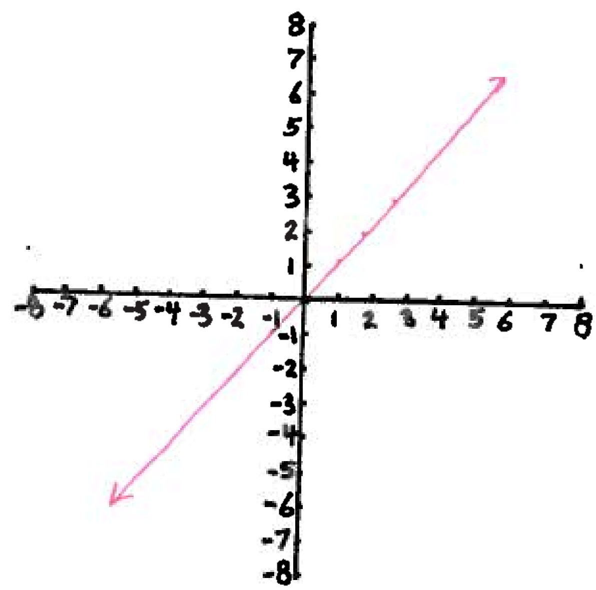


4) Graph of $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^n$ where $n \in \mathbb{N}$, $n \geq 3$, and n is odd looks similar to graph of $h(x) = x^3$



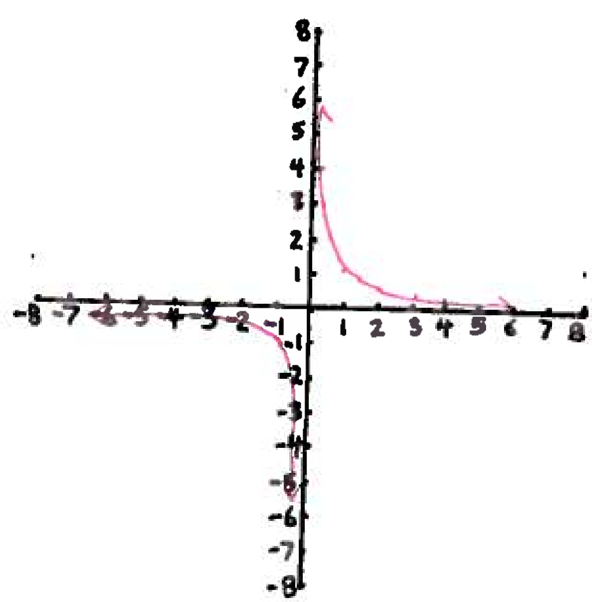
5) $id: \mathbb{R} \to \mathbb{R}$

x	$id(x)$	$(x, id(x))$
-2	-2	(-2, -2)
-1	-1	(-1, -1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)

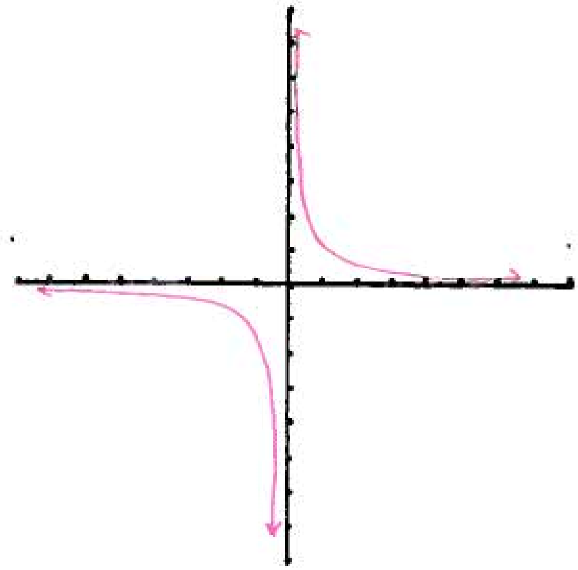


6) $g: \mathbb{R} - \{0\} \to \mathbb{R}$, $g(x) = 1/x$

x	$g(x)$	$(x, g(x))$
1	$\frac{1}{1} = 1$	(1, 1)
2	$\frac{1}{2} = \frac{1}{2}$	$(2, \frac{1}{2})$
3	$1/3 = 1/3$	$(3, 1/3)$
4	$1/4 = 1/4$	$(4, 1/4)$
$1/2$	$1/(1/2) = 2$	$(\frac{1}{2}, 2)$
$1/3$	$\frac{1}{1/3} = 3$	$(\frac{1}{3}, 3)$
$1/4$	$\frac{1}{1/4} = 4$	$(\frac{1}{4}, 4)$

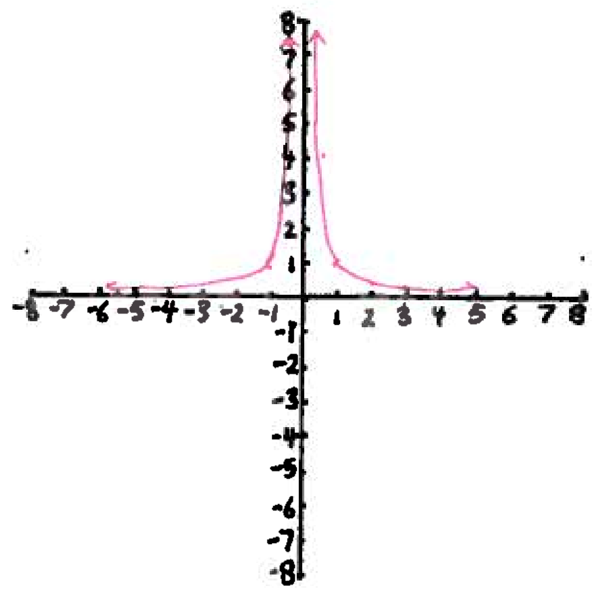


7) Graph of
$h: \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$, $h(x) = \frac{1}{x^n}$
where $n \in \mathbb{N}$ is odd looks
similar to graph of $g(x) = \frac{1}{x}$

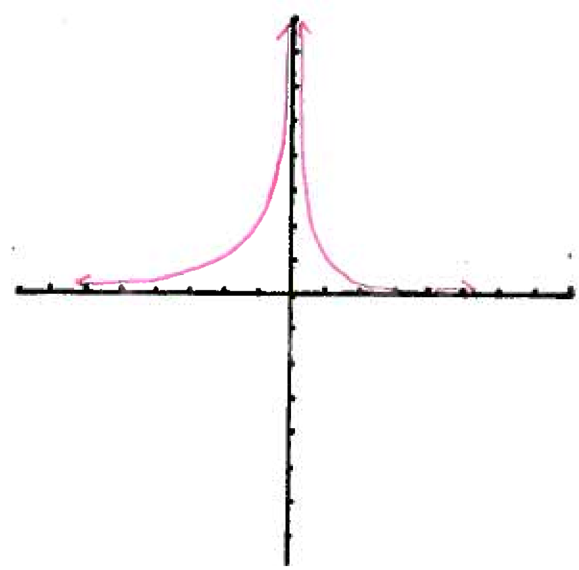


8.) $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R}$, $f(x) = \frac{1}{x^2}$

x	$f(x)$	$(x, f(x))$
1	$\frac{1}{1^2} = 1$	$(1, 1)$
2	$\frac{1}{2^2} = \frac{1}{4}$	$(2, \frac{1}{4})$
3	$\frac{1}{3^2} = \frac{1}{9}$	$(3, \frac{1}{9})$
4	$\frac{1}{4^2} = \frac{1}{16}$	$(4, \frac{1}{16})$
$\frac{1}{2}$	$\frac{1}{(\frac{1}{2})^2} = 4$	$(\frac{1}{2}, 4)$
$\frac{1}{3}$	$\frac{1}{(\frac{1}{3})^2} = 9$	$(\frac{1}{3}, 9)$
$\frac{1}{4}$	$\frac{1}{(\frac{1}{4})^2} = 16$	$(\frac{1}{4}, 16)$

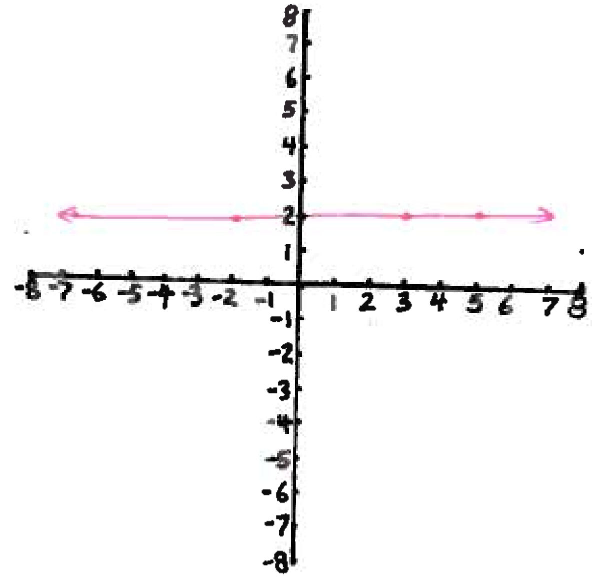


9.) Graph of $g: \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$, $g(x) = \frac{1}{x^n}$
where $n \in \mathbb{N}$ is even looks
similar to graph of $f(x) = \frac{1}{x^2}$



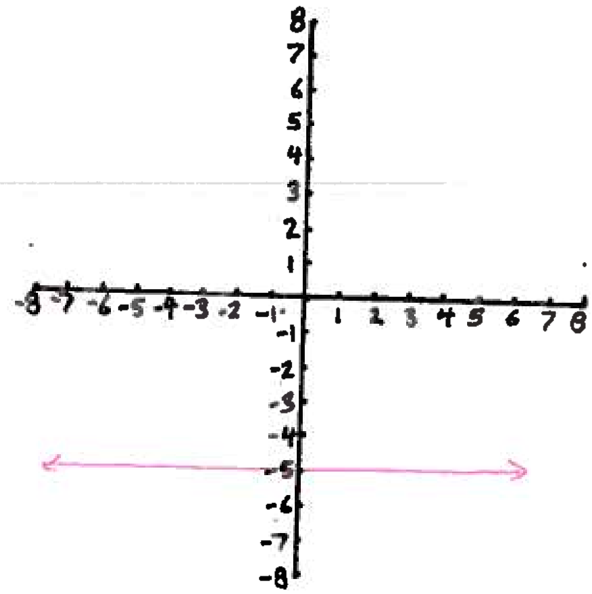
10.) $h : \mathbb{R} \to \mathbb{R}$, $h(x) = 2$

x	$h(x)$	$(x, h(x))$
-7	2	(-7, 2)
-2	2	(-2, 2)
0	2	(0, 2)
3	2	(3, 2)
5	2	(5, 2)

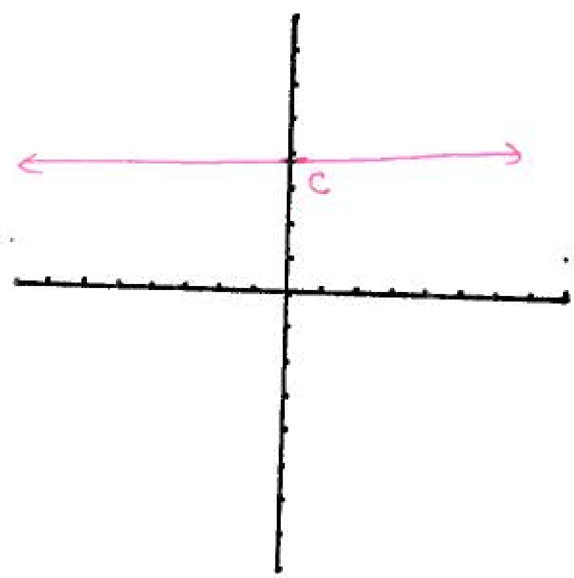


11.) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = -5$

x	$f(x)$	$(x, f(x))$
-6	-5	(-6, -5)
-1	-5	(-1, -5)
0	-5	(0, -5)
2	-5	(2, -5)
8	-5	(8, -5)



12.) Graph of $g : \mathbb{R} \to \mathbb{R}$, $g(x) = c$ (where $c \in \mathbb{R}$ is a constant so that g is a constant function) looks similar to the graphs of $h(x) = 2$ and $f(x) = -5$.



Graph Transformations

On this page, $d > 0$ and $c > 1$. $f(x)$ is a function whose graph we know.

New function	How points in graph of $f(x)$ become points in graph of new function.	How the overall picture changes
$f(x)+d$	$(a,b) \longmapsto (a,b+d)$	shift up by d
$f(x)-d$	$(a,b) \longmapsto (a,b-d)$	shift down by d
$cf(x)$	$(a,b) \longmapsto (a,cb)$	stretch vertically by c
$\frac{1}{c}f(x)$	$(a,b) \longmapsto (a,\frac{b}{c})$	shrink vertically by c
$-f(x)$	$(a,b) \longmapsto (a,-b)$	flip over x-axis

↑ change occurs "after f"

↑ change occurs in second coordinate

↑ change is vertical

change occurs "before f" ↓

change occurs in first coordinate ↓

change is horizontal ↓

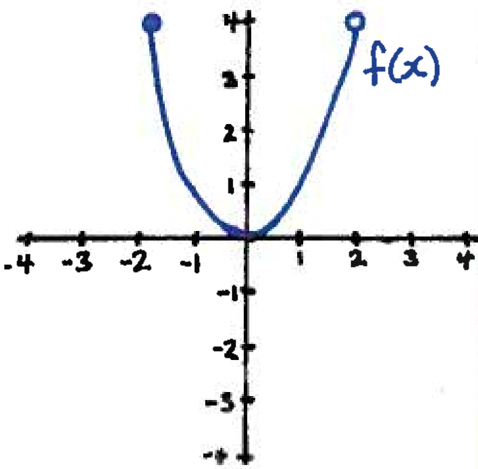
$f(x+d)$	$(a,b) \longmapsto (a-d,b)$	shift left by d
$f(x-d)$	$(a,b) \longmapsto (a+d,b)$	shift right by d
$f(cx)$	$(a,b) \longmapsto (\frac{a}{c},b)$	shrink horizontally by c
$f(\frac{x}{c})$	$(a,b) \longmapsto (ca,b)$	stretch horizontally by c
$f(-x)$	$(a,b) \longmapsto (-a,b)$	flip over y-axis

change in these columns may seem counterintuitive for this second grouping of five functions.

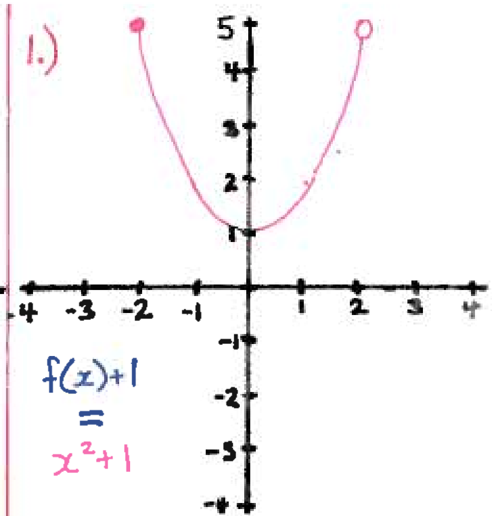
On this page,

$f: [-2,2) \longrightarrow \mathbb{R}$

$f(x) = x^2$

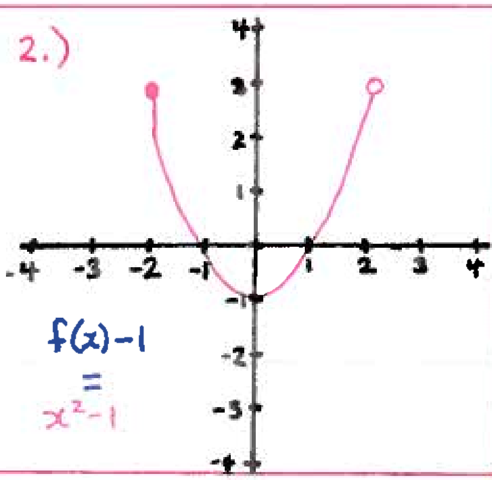


1.)



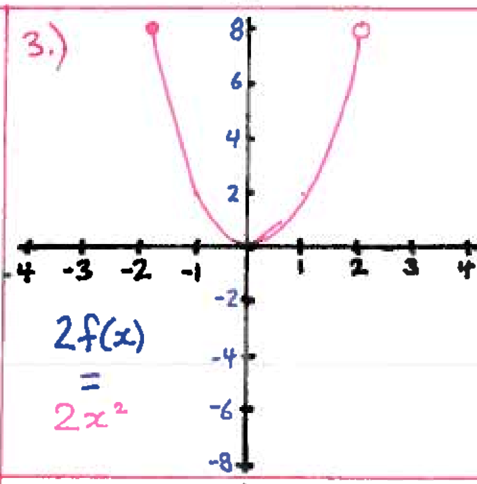
$f(x)+1 = x^2+1$

2.)



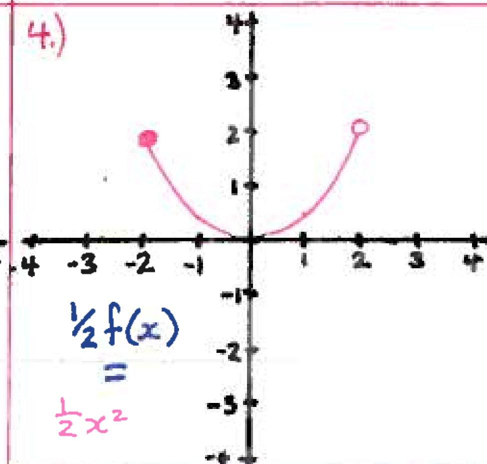
$f(x)-1 = x^2-1$

3.)



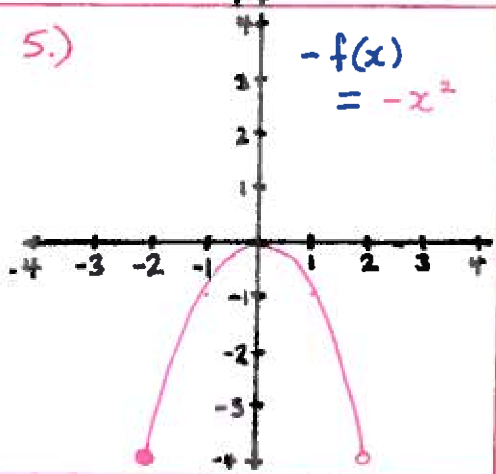
$2f(x) = 2x^2$

4.)



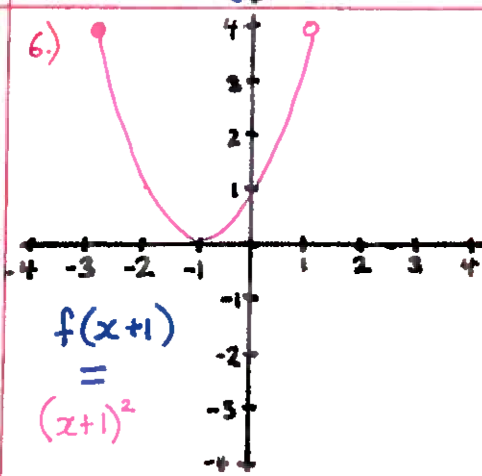
$\frac{1}{2}f(x) = \frac{1}{2}x^2$

5.)



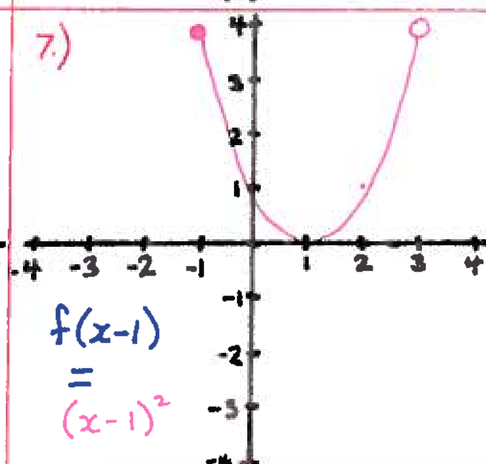
$-f(x) = -x^2$

6.)



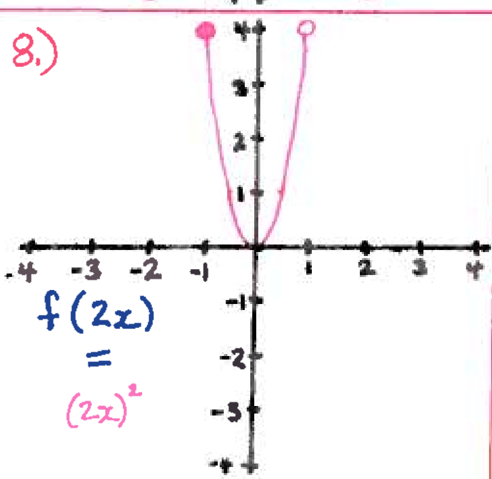
$f(x+1) = (x+1)^2$

7.)

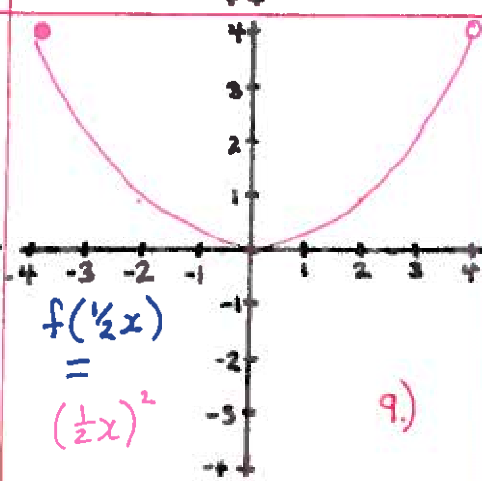


$f(x-1) = (x-1)^2$

8.)



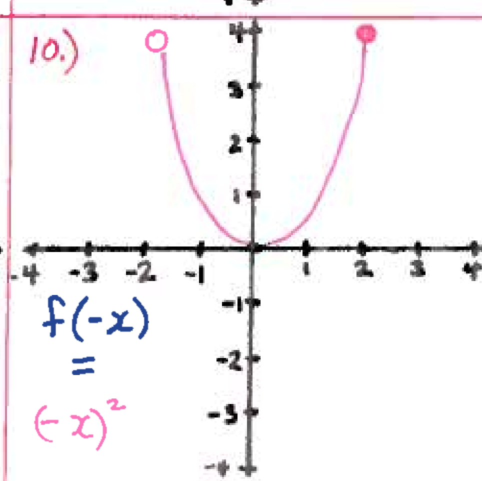
$f(2x) = (2x)^2$



$f(\frac{1}{2}x) = (\frac{1}{2}x)^2$

9.)

10.)



$f(-x) = (-x)^2$