Answer all questions below. Every question is worth the same number of points. You may use the available space for work, but must write ALL answers on the answer sheet. No cell phones, calculators, or notes are allowed during the exam. These rules will be strictly enforced.

These questions are intended as a tool to help you begin reviewing for the final exam. Not every possible problem or type of problem will be covered, but you can look up similar problems in your homework, textbook, or previous review exercises if you want more practice with a particular topic.

**Sets and Numbers**

1. Consider the sets $\mathbb{R}, \mathbb{N}, \mathbb{Q}, \mathbb{Z}$.
   (a) What is the smallest of these sets that contains $\pi$?
   (b) What is the smallest of these sets that contains 0?
2. Is it true that $[0, 17) \subseteq (0, 17]$?
3. Is $\sqrt{2} \notin \mathbb{Q}$?

**Sequences, Sums and Series**

4. Is the sequence $4, 8, 12, 16, \ldots$ arithmetic, geometric, or neither?
5. Is the sequence $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}$ arithmetic, geometric, or neither?
6. Find the 54th term in the arithmetic sequence $1, 6, 11, 16, \ldots$
7. Find the sum $\sum_{i=1}^{4}(i^2 - 3)$.
8. Find the sum $\sum_{i=1}^{500} 4$.
9. Find the sum of the first 101 terms of the sequence $5, 7, 9, 11, 13, \ldots$
10. Find the value of the series $100 + 25 + \frac{25}{4} + \frac{25}{16} + \ldots$

**Counting**

11. You are designing a house for yourself to live in. You can choose the house to be made out of wood, brick, or metal. The roof can be wood shingles, asphalt shingles, or tin. You can paint the house brown, red, yellow, or green. You can choose to have two, three, four, or five bedrooms. How many possibilities are there for the design of your house?

12. You are in an ice cream shop. There are 31 flavors of ice cream. You are building a sundae with a scoop of ice cream on the bottom, a different flavor on top of that, and a third different flavor on top of that. If it matters to you which flavor is on the bottom, in the middle, and on the top, then how many different sundaes can you make?

13. You are on a boat with 200 people. The boat is sinking and there is only one life raft with 8 seats. How many different options are there for which 8 people to put in the raft?

14. Your baseball team is choosing a batting order. There are 27 players on the team, and you must determine an order in which the players will get to bat. How many different batting orders are possible?

15. Write out $(x-2y)^5$ so that your answer doesn't include any numbers that look like $\binom{n}{k}$

16. Write $\binom{12}{9}$ as a natural number in standard form.

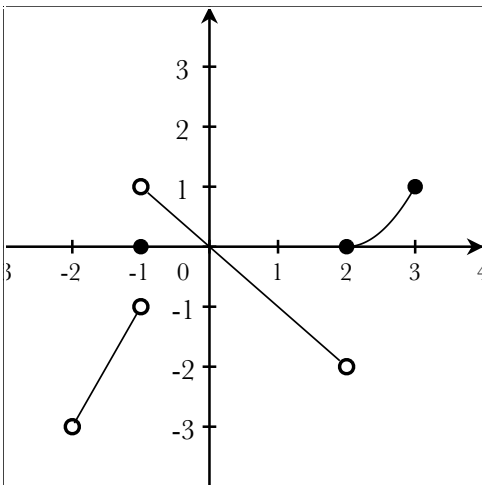
**Functions & Graphs**

17. What is the implied domain of $f(x) = \frac{2}{3x-8}$?

18. What is the implied domain of $g(x) = \frac{(x-2)}{(x-3)(x+1)} + \sqrt[3]{x^2-4}$?

19. What is the implied domain of $h(x) = \sqrt[4]{(3x-15)}$?

20. Below is the graph of a function. Answer the following questions:




(a) What is $f(2)$?

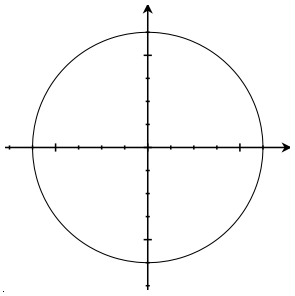
(b) What is the domain of $f$?

(c) What is the range of $f$?

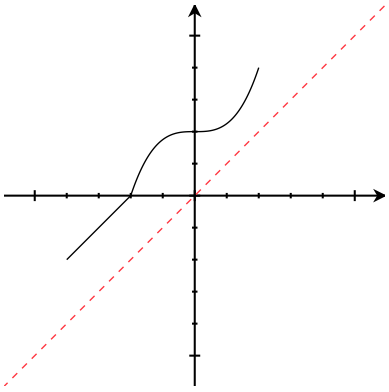
(d) What are the $x$-intercepts of the graph?

(e) What are the $y$-intercepts of the graph?

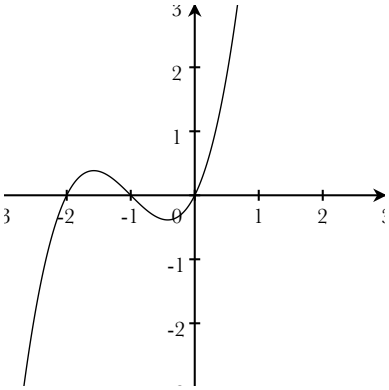
21. Is the following picture the graph of a function?



22. Below is the graph of an invertible function $h(x)$. Graph the inverse function, $h^{-1}(x)$. (The line $y = x$ is shown in red.)



23. Below is the graph of a function $f(x)$. Does $f$ have an inverse?




24. Find the inverse of $g(x) = \frac{2}{3x-8}$.

25. If $f(x) = 3x^2 + 1$ and $g(x) = x + 2$ find $f \circ g(x)$.

26. Is it sometimes, always, or never true that $f \circ g(x) = g \circ f(x)$?

**Polynomials**

27. Find a root of the polynomial $2x^3 + 3x^2 - 3x - 2$.

28. Completely factor $2x^3 - 8x^2 + 2x + 12$ given that $-1$ is a root.

29. Find the quotient: $\dfrac{5x^3 + 8x^2 - 5x + 7}{x^2 + 2x + 1}$

30. What is the leading term of the polynomial $12x(2x^2 - 3x + 11)(-x^7 + 4x^5 - 2x + 11)$?

31. True or false: the polynomial $13x^7 + 11x^5 - 6x^2 + 1$ has 13 roots.

32. Complete the square: Write $-2x^2 + 4x + 1$ in the form $\alpha(x + \beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

33. Completely factor $-2x^2 + 4x + 1$.

34. How many roots does $-x^2 + 3x - 5$ have?

**Exponentials and Logarithms**

35. Rewrite $\log_4(x)$ using logarithms with base 10.

36. Write $\log_4(8)$ as a rational number. (Hint: Use the change of base formula with an appropriate base.)

37. What is the greatest integer less than $\log_{10}(9999)$?

38. Rewrite $3^{x+2} = 17$ as a logarithmic equation.

39. Rewrite $\log_{10}(x^2 + 7x + 1) = 4$ as an exponential equation.

40. You owe the bank \$1000. If the bank charges 12% annual interest, how much will you owe after 3 years?

**Solving Equations:** Solve for $x$.

41. $(6 - x)^3 + 5 = 32$

42. $\log_5(x) = 3$

43. $\log_5(x + 1) + \log_5(2x) = 3\log_5(2) + 1$

44. $(\frac{1}{4})^x = 32$

45. $8^{\log_4(x)} = 4$

**Linear Algebra**

46. Solve the system of equations:

$$\begin{cases} 2x + 3y = 8 \\ -x + y = 1 \end{cases}$$

47. Is $x = 1, y = 2, x = 3$ a solution to the following system of equations?

$$\begin{cases} x + y + z = 6 \\ 2x + 4y - z = 7 \\ x - z = 0 \end{cases}$$

48. Find the inverse of $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$.

49. Compute the following matrices:

(a) $\begin{pmatrix} 1 & 3 & 4 \\ 7 & -2 & 1 \\ 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$

50. Calculate $\det\left[\begin{pmatrix} 1 & 3 \\ 4 & 0 \end{pmatrix}\right]$.

51. Given that

$$\begin{pmatrix} 1 & 1 & -3 \\ -3 & -4 & 10 \\ 2 & 2 & -7 \end{pmatrix}^{-1} = \begin{pmatrix} 8 & 1 & -2 \\ -1 & -1 & -1 \\ 2 & 0 & -1 \end{pmatrix},$$

solve the system of equations

$$\begin{cases} x + y - 3z = 5 \\ -3x - 4y + 10z = 0 \\ 2x + 2y - 7z = -1 \end{cases}$$

**Graphing**

52. Graph the following functions.

(a) $id(x)$

(b) $\frac{1}{x}$

(c) $\frac{1}{x^2}$

(d) $x^2$

(e) $x^3$

(f) $f(x) = 5$

(g) $\sqrt[3]{x}$

(h) $\sqrt{x}$

(i) $(\frac{1}{2})^x$

(j) $\log_3(x)$

(k) $10^x$

(l) $\log_{\frac{1}{4}}$

53. Graph the following functions and label all $x$- and $y$-intercepts, vertical and horizontal asymptotes (if there are any).

(a) $f(x) = -2\sqrt[3]{x+4}$

(b) $g(x) = \begin{cases} 2x-1 & \text{if } x \in (-\infty, -1] \\ -3 & \text{if } x \in (-1, 3] \\ \frac{1}{(x-4)^6} & \text{if } x \in (3, \infty) \end{cases}$

(c) $h(x) = \log_3(x+1) - 2$

(d) $F(x) = \pi^{x+2}$

(e) $G(x) = \log_{\frac{1}{2}}(x) + 2$

(f) $H(x) = 2^{2x} - 4$

(g) $p(x) = -3(x-1)(x+2)(x+2)(x+\frac{2}{3})$

(h) $r(x) = \dfrac{-2(x-4)(x+2)(x^2+6)}{3(x-1)(x^2+x+2)}$

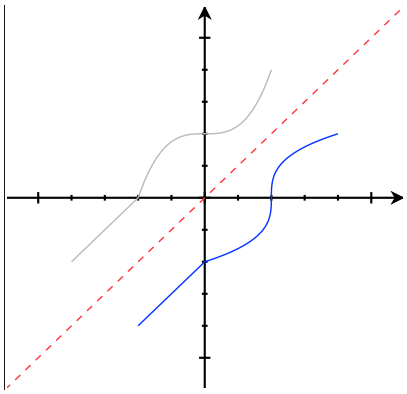
54. Graph $p(x) = -2(x+1)^2 + 2$ and label the vertex.

55. What is the slope of the straight line that passes through the points $(-1, 3)$ and $(7, 5)$?

**Answers**

1. (a) $\mathbb{R}$
   (b) $\mathbb{Z}$
2. False: $0 \in [0, 17)$ but $0 \notin (0, 17]$
3. True: $\sqrt{2}$ is not a rational number.
4. Arithmetic $(d = 4)$
5. Geometric $(= -\frac{1}{2})$
6. $a_{54} = 1 + (54 - 1)(5)$
7. 18
8. $4(500) = 2000$
9. $\frac{210(101)}{2}$
10. $\frac{100}{1-\frac{1}{4}} = \frac{400}{3}$
11. $3(3)(4)(4)$
12. $\frac{31!}{28!} = 31(30)(29)$
13. $\binom{200}{8} = \frac{200!}{(192!)(8!)}$
14. 27!
15. $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$
16. 220
17. $\mathbb{R} - \{\frac{8}{3}\}$
18. $\mathbb{R} - \{3, -1\}$
19. $[5, \infty)$
20. (a) $f(2) = 0$
    (b) $(-2, 3]$
    (c) $(-3, 1]$
    (d) $-1, 2$
    (e) 0
21. Not a function.

22. The inverse is shown in blue, the original graph in gray:



23. No (Doesn't pass the horizontal line test, so it is not one-to-one.)

24. $g^{-1}(y) = \frac{2+8y}{3y}$

25. $f \circ g(x) = 3(x+2)^2 + 1$

26. Sometimes: For example, $f \circ f^{-1}(x) = f^{-1} \circ f(x) = id(x)$, but for the two functions in the previous question, $f \circ g \neq g \circ f$.

27. $x = 1$ is a root. ($x = -2$ is also a root)

28. $2(x+1)^2(x-6)$

29. $5x - 2 + \frac{-6x+9}{x^2+2x+1}$

30. $-24x^10$

31. False: degree $n$ has at most $n$ roots.

32. $-2(x + \frac{4}{-4})^2 + 1 - \frac{4^2}{-8}$

33. $-2(x - \frac{-4+\sqrt{24}}{-4})(x - \frac{-4-\sqrt{24}}{-4}$

34. 0 roots

35. $\frac{\log_{10}(x)}{\log_{10}(4)}$

36. $\frac{3}{2}$

37. 3

38. $\log_3(17) = x + 2$

39. $10^4 = x^2 - 7x + 1$

40. $1000(0.12)^3$

41. $x = 3$

42. $x = 5^3$

43. $x = \frac{-2+\sqrt{324}}{4}, x = \frac{-2-\sqrt{324}}{4}$

44. $\log_{\frac{1}{4}}(32)$

45. $4^{\frac{2}{3}}$ or $8^4 9$ (these are equivalent, you might see one or the other depending on how you solve.)

46. $x = 1, y = 2$

47. No

48. $\begin{pmatrix} \frac{1}{3} & \frac{-2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$.

49. (a) $\begin{pmatrix} 19 \\ 6 \\ 28 \end{pmatrix}$

    (b) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}$

50. $-12$

51. $x = 42, y = -4, z = 11$