Math 1050 Exam 2 March 4, 2016

Answer all questions below. You may use the available space for work, but must write ALL answers in the answer booklet. No cell phones, calculators, notes, or talking are allowed during the exam.

Name: Key UID: _____

(Answers on last 3 pages)

Tip: If you are stuck on a question, move on and come back to it later.

True or False

For #1-?? decide whether the statement is true or false. Write the word "True" or the word "False" on your answer sheet. Ambiguous answers will receive no credit. ($\frac{1}{2}$ point each.)

1. $\pi \in \mathbb{Q}$ F
2. $2.999 \in (0,3)$ T
3. $\mathbb{Z} \subseteq \mathbb{N}$ F
4. $-x^3 + 17x^2 + 2x - 3$ has 5 roots. F (degree 3)
5. -1 is a root of $2x^3 - 3x^2 - 7x - 2$. $-2+3+7-2 = 10-4=6$ False
6. $\sqrt[n]{x+y} = \sqrt[n]{x} + \sqrt[n]{y}$ F
7. $(x+y)^n = x^n + y^n$ F
8. $a(b+c) = ab + ac$ T
9. $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{bd}$ F
10. 0 is always in the implied domain of $f(x) = \sqrt[n]{x}$. T

Inverse Functions and n^{th} Roots

11. Solve for x: $\sqrt[4]{x+7} + 3 = 2$

$\sqrt[4]{x+7} = 2-3 = -1$

$\left(\sqrt[4]{x+7}\right)^4 = (-1)^4$

$x+7 = 1$

$x = -6$

12. If $f(x)$ is an invertible function and $f(14) = 7$, then what is $f^{-1}(7)$? 14

13. Find the inverse of $g(x) = (x+3)^3$.

$y = (x+3)^3$

$\sqrt[3]{y} = \sqrt[3]{(x+3)^3}$

$\sqrt[3]{y} = x+3$

$\sqrt[3]{y} - 3 = x$

$g^{-1}(y) = \sqrt[3]{y} - 3$

14. (2 points) What is the implied domain of $h(x) = \sqrt[12]{2x+7} - 12$? Write your answer using interval notation.

$2x + 7 \geq 0$ (even root)

$2x \geq -7$

$x \geq \frac{-7}{2}$

$\left[-\frac{7}{2}, \infty\right)$

Polynomials

15. Find the quotient: $\dfrac{4x^3 - 6x^2 - 20x + 8}{x^2 - 3}$

$$4x - 6 + \frac{-8x - 10}{x^2 - 3}$$

$$\begin{array}{r} 4x - 6 \\ x^2 - 3 \overline{)\,4x^3 - 6x^2 - 20x + 8} \\ -(4x^3 \qquad - 12x) \\ \hline -6x^2 - 8x + 8 \\ -(-6x^2 \qquad + 18) \\ \hline -8x - 10 \end{array}$$

16. Find the quotient: $\dfrac{4x^3 - 5x + 6}{x - 2}$

$$4x^2 + 8x + 11 + \frac{28}{x-2}$$

$$\begin{array}{r} 4x^2 + 8x + 11 \\ x - 2 \overline{)\,4x^3 + 0x^2 - 5x + 6} \\ -(4x^3 - 8x^2) \\ \hline 8x^2 - 5x + 6 \\ -(8x^2 - 16x) \\ \hline 11x + 6 \\ -(11x - 22) \\ \hline 28 \end{array}$$

$$\begin{array}{r|cccc} 2 & 4 & 0 & -5 & 6 \\ & & 8 & 16 & 22 \\ \hline & 4 & 8 & 11 & 28 \end{array}$$

$$4x^2 + 8x + 11 + \frac{28}{x-2}$$

17. What is the degree of the polynomial $p(x) = -2(x+3)(x-4)(x+1)(x^2+1)(x^2+3)$?

Leading term: $-2(x)(x)(x)(x^2)(x^2)$

$= -2x^7$

Degree: 7

18. What is the slope of the line in $\mathbb{R}^2$ that passes through the points $(1,3)$ and $(-1,2)$?

$$\text{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3-2}{1-(-1)} = \frac{1}{2}$$

19. How many roots does $p(x) = 5x^2 - 2x - 1$ have?

discriminant: $b^2 - 4ac = (-2)^2 - 4(-1)(5)$

$= 4 + 20 = 24 > 0$

2 roots

20. Find all roots of $q(x) = -2x^2 + 5x + 1$. If there are no roots, write "none."

$b^2 - 4ac = 5^2 - 4(-2)(1) = 25 - (-8) = 33$: 2 roots

Quadratic Formula:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-5 + \sqrt{33}}{-4} \text{ and } \frac{-5 - \sqrt{33}}{-4}$$

21. Find a root of the polynomial $g(x) = -2x^3 + 11x^2 - 13x - 2$.

Check factors of -2: $1, -1, -2, 2$

$g(1) = -2 + 11 - 13 - 2 = 11 - 17 = -6$

$g(-1) = -2(-1)^3 + 11(-1)^2 - 13(-1) - 2$

$= 2 + 11 + 13 - 2 \neq 0$

$g(2) = -2(8) + 11(4) - 26 - 2$

$= -16 + 44 - 26 - 2$

$= 44 - 42 - 2$

$= 0$

2 is a root.

22. Complete the square: Write $-3x^2+6x+7$ in the form $\alpha(x+\beta)^2+\gamma$, where $\alpha, \beta, \gamma \in \mathbb{R}$.

$$ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

$$-3x^2+6x+7 = -3\left(x+\frac{6}{-6}\right)^2+7-\frac{6^2}{-12}$$

$$= -3(x-1)^2+7+\frac{36}{12}$$

$$= -3(x-1)^2+7+3$$

$$= -3(x-1)^2+10$$

Factoring Polynomials

For #??- ??, completely factor the following polynomials. Your answer should be a product of a constant and maybe some linear and quadratic polynomials that have leading coefficients equal to 1, such that any of the quadratics in the product have no roots.

23. Completely factor $10x+5$.

$$10\left(x+\frac{5}{10}\right)$$

$$= 10\left(x+\frac{1}{2}\right)$$

24. Completely factor $2x^2-5x+3$.

discriminant: $b^2-4ac = (-5)^2-4(2)(3) = 25-24 = 1$

2 roots: $\frac{5+\sqrt{1}}{4}=\frac{3}{2}$ and $\frac{5-\sqrt{1}}{4}=1$

$$2\left(x-\frac{3}{2}\right)(x-1)$$

25. (2 points) Completely factor $4x^3 - 4x^2 - 20x - 12$, given that 3 is a root.

$4x^3 - 4x^2 - 20x - 12$

$(x-3)$ $\quad$ $4x^2+8x+4$

4 $\quad$ $(x+1)^2$

3	4	-4	-20	-12
		12	24	12
	4	8	4	0

Factor $4x^2+8x+4$:

discriminant:

$b^2-4ac = 8^2-4(4)(4)$

$= 64-64$

$= 0$

1 root: $\frac{-8+\sqrt{0}}{8} = -1$

$4(x+1)^2$

$4(x-3)(x+1)^2$

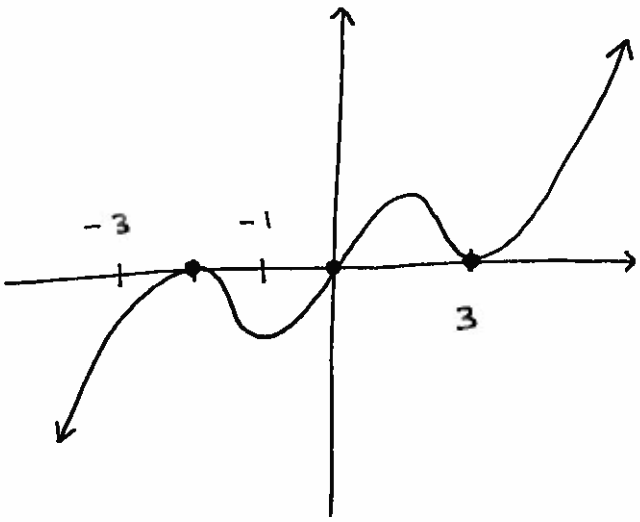
26. Write the following as a single fraction: $\dfrac{2x+\frac{5}{x^2}}{\frac{6x^4}{x}+\frac{5x}{x}}$

$$= \frac{\frac{2x^3}{x^2}+\frac{5}{x^2}}{\frac{6x^4+5x}{x}} = \frac{\frac{2x^3+5}{x^2}}{\frac{6x^4+5x}{x}}$$

$$= \frac{2x^3+5}{x^2} \cdot \frac{x}{6x^4+5x} = \frac{2x^3+5}{(6x^4+5x)x} = \boxed{\frac{2x^3+5}{6x^5+5x^2}}$$

Graphing

27. The graph of a polynomial $p(x)$ is shown below. List all of the monic linear factors of $p(x)$.



Roots: 3, 0, -2

$(x-3), (x-0), (x+2)$

Graph the following functions in the space provided in your answer booklet. Label any indicated information.

28. (2 points) $f(x) = 2(x-2)^2+1$ (Label the vertex.)

29. (2 points) $g(x) = \sqrt{-x}$ (Label the y-intercept.)

30. (2 points) $h(x) = \sqrt[3]{x+5}$ (Label the x-intercept.)

31. (2 points) $p(x) = -4x+8$ (Label the x- and y-intercepts.)

32. (3 points) $q(x) = 3(x^2-x+1)(x-2)^2(x+5)$ (Label the x- and y-intercepts.)

33. (0 points) How are things going in the course? What is working well and what isn't?

You may use this space for scratchwork. This page will not be graded.

Name: Key UID: _____

½ pt each

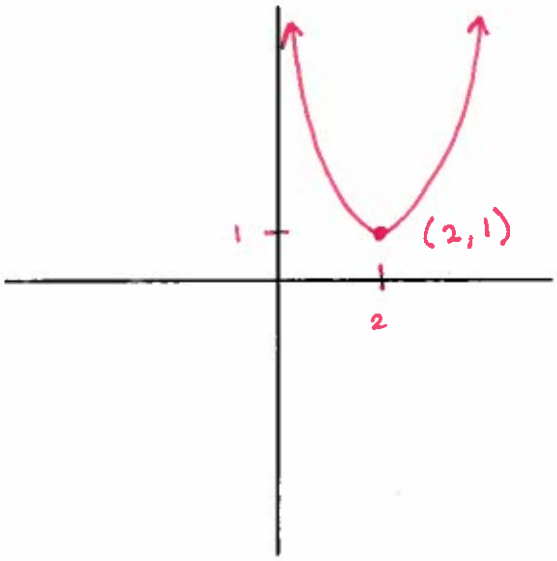
1. False
2. True
3. False
4. False
5. False
6. False
7. False
8. True
9. False
10. True
11. -6
12. 14
13. $g^{-1}(y) = \sqrt[3]{y} - 3$

14. $[-7/2, \infty)$
15. $4x - 6 + \frac{-8x - 10}{x^2 - 3}$
16. $4x^2 + 8x + 11 + \frac{28}{x-2}$
17. 7
18. $\frac{1}{2}$
19. 2 roots
20. $\frac{-5+\sqrt{33}}{-4}$ and $\frac{-5-\sqrt{33}}{-4}$
21. 2
22. $-3(x-1)^2 + 10$
23. $10\left(x + \frac{1}{2}\right)$
24. $2\left(x - \frac{3}{2}\right)(x-1)$
25. $4(x-3)(x+1)^2$
26. $\frac{2x^3+5}{6x^5+5x^2}$
27. $(x-3)$, $(x-0)$ or x, $(x+2)$

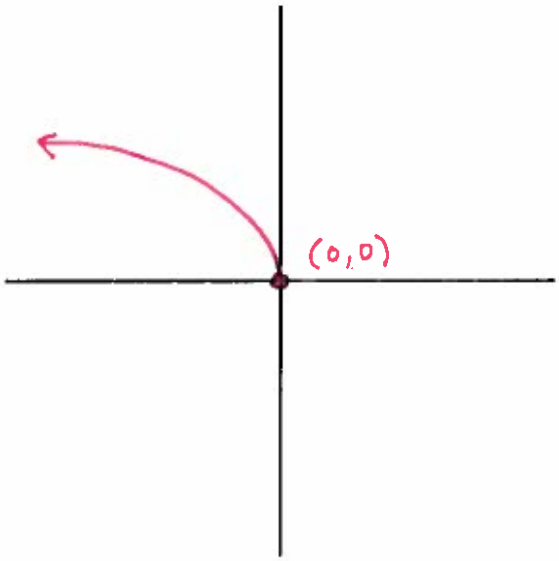
* See Section Roots & Factors, #7 & 8 for other examples like #27

Graph the following functions. Label any information listed in parenthesis.

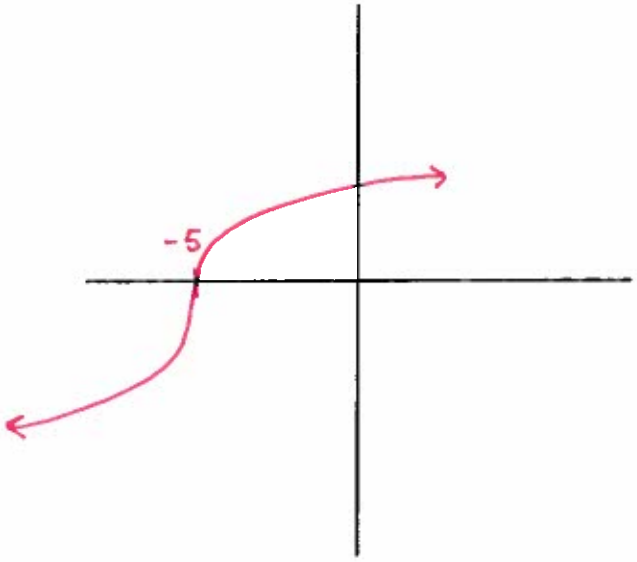
27. (2 points) $f(x) = 2(x-2)^2 + 1$ (Label: vertex.)



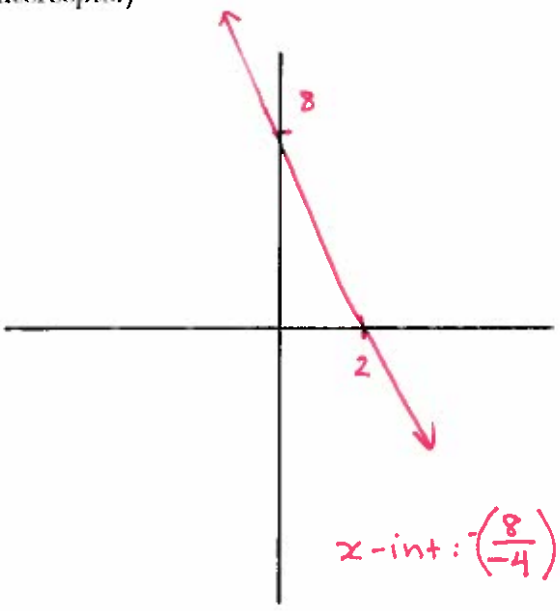
28. (2 points) $g(x) = \sqrt{-x}$ (Label: y-intercept.)



29. (2 points) $h(x) = \sqrt[3]{x+5}$ (Label: x-intercept.)

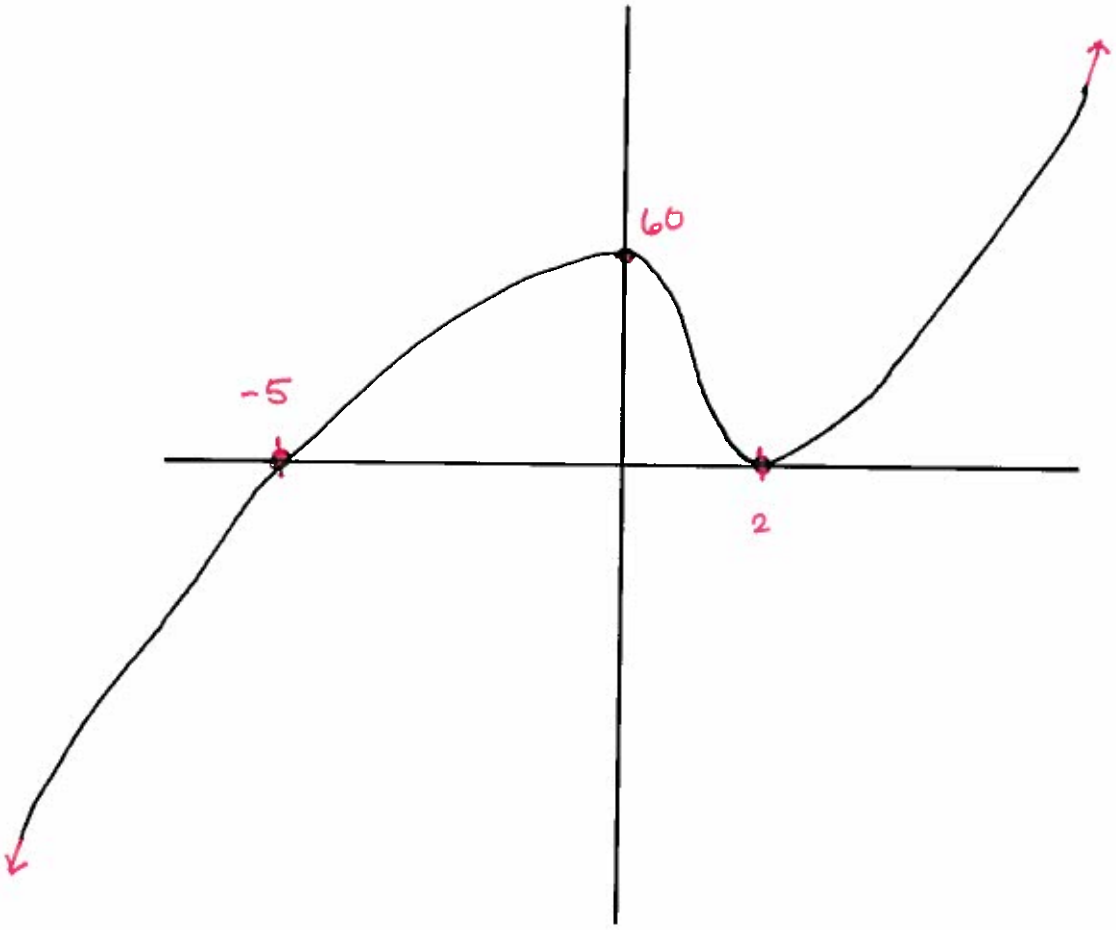


30. (2 points) $p(x) = -4x + 8$ (Label: both intercepts.)



x-int: $-\left(\frac{8}{-4}\right) = 2$

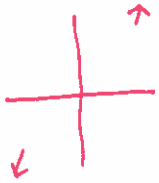
31. (3 points) $q(x) = 3(x^2 - x + 1)(x - 2)^2(x + 5)$ (Label: both intercepts.)



$q(0) = 3(1)(-2)^2(5) = 12 \cdot 5 = 60$

Leading term: $3x^2(x)^2(x) = 3x^5$

Far right and far left:



("connect the dots" is in black. All other info in red.)