Name: Key UID: _____

1. Geometric
2. Arithmetic
3. geometric
4. $8 \cdot \left(\frac{5}{8}\right)^{200}$
5. $\frac{-5}{1-(-\frac{1}{4})}$ (=) OR -4
6. $25(149)$
7. $\frac{64}{3}$
8. 24
9. $\frac{100!}{97!}$ (OR) $= 100 \cdot 99 \cdot 98$
10. 41

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11. ~~$(2x-y)^4 =$~~ $(x-2y)^5 =$

$$x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$$

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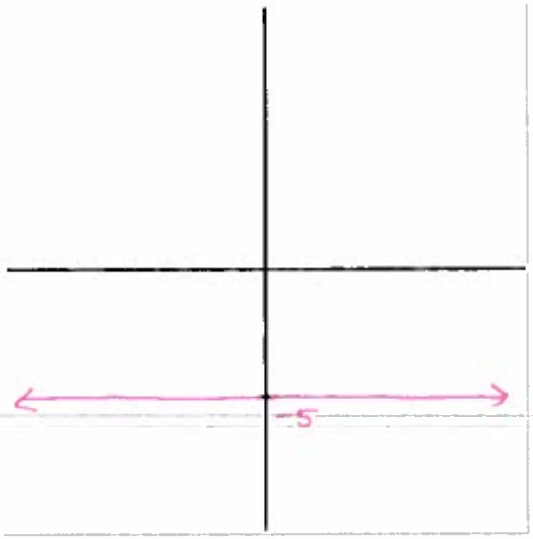
12. $\mathbb{R} - \{-1\}$
13. $\mathbb{R} - \{0\}$
14. $\mathbb{R}$
15. 8
16. 110
17. $(0,1)$
18. $(-1,0)$
19. $[-3,4)$
20. $[0,4]$
21. $[-3,-1)$ and $(-1,4)$ (OR: "$[-3,4) - \{-1$
22. $f^{-1}(y) = \frac{-y}{y-1}$
23. $f^{-1}(y) = \frac{y-3}{2}$
24. False

25. (3 points) $f(x) = -5$

Domain: $\mathbb{R}$

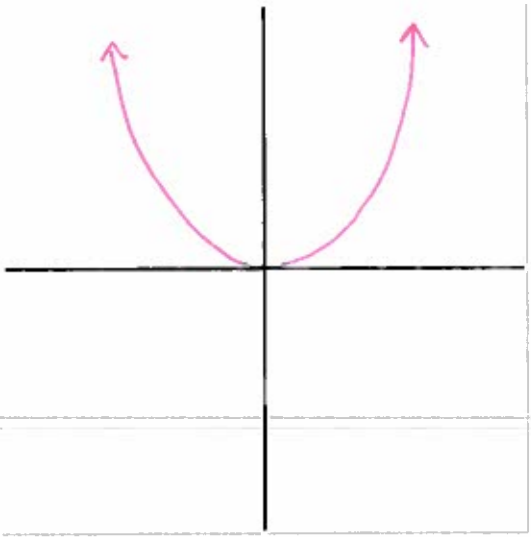
Range: $\{-5\}$



26. (3 points) $g(x) = x^2$

Domain: $\mathbb{R}$

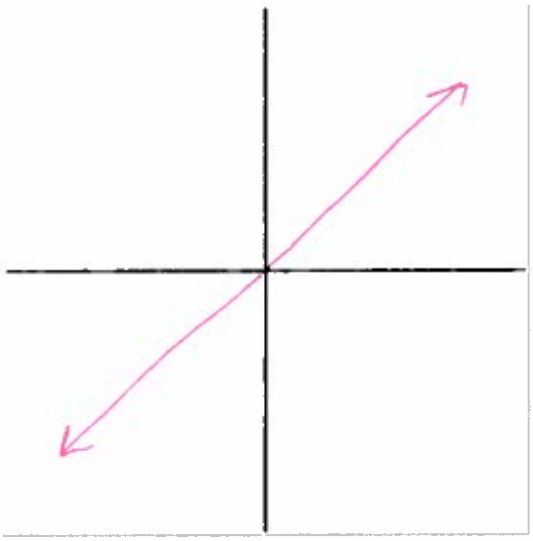
Range: $[0, \infty)$



27. (3 points) $h(x) = x$

Domain: $\mathbb{R}$

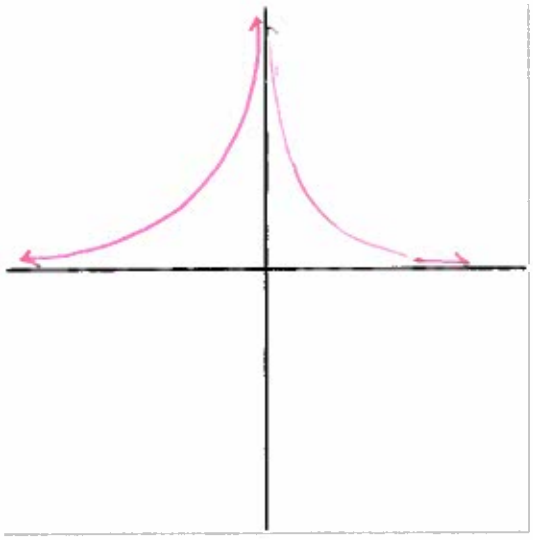
Range: $\mathbb{R}$



28. (3 points) $F(x) = \frac{1}{x^2}$

Domain: $\mathbb{R} - \{0\}$

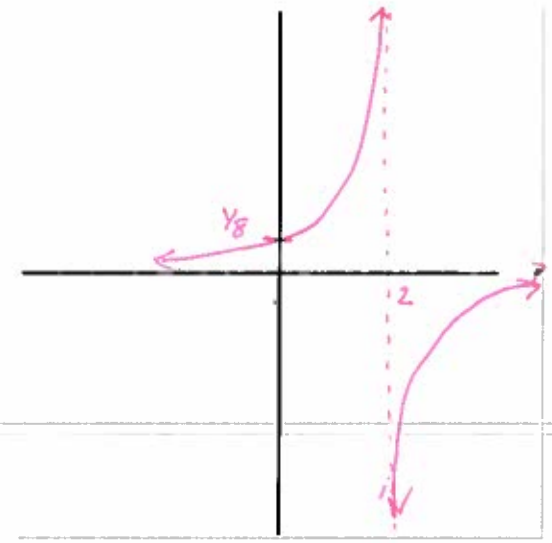
Range: $(0, \infty)$



29. (4 points) $G(x) = -\frac{1}{(x-2)^3}$

Domain: $\mathbb{R} - \{2\}$

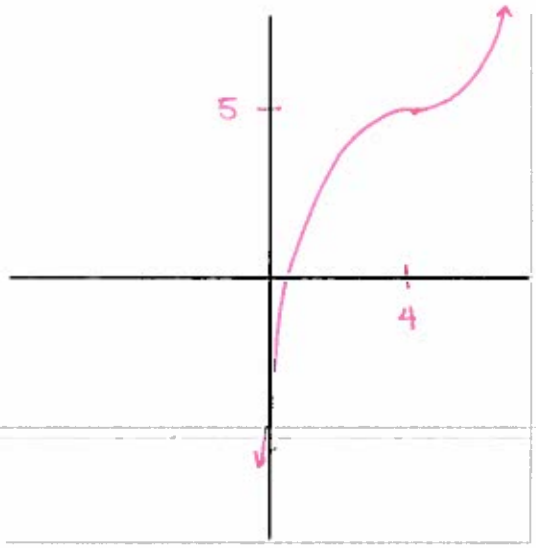
Range: $\mathbb{R} - \{0\}$



30. (4 points) $f(x) = (x-4)^3 + 5$

Domain: $\mathbb{R}$

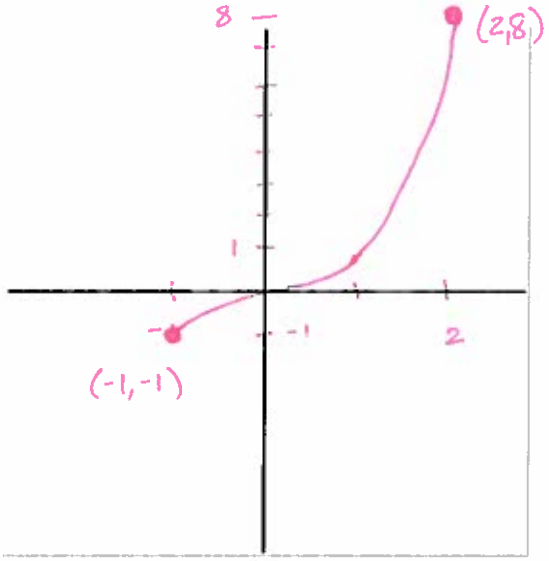
Range:



31. (4 points) $g : [-1, 2] \to \mathbb{R}$ where $g(x) = x^3$

Domain: $[-1, 2]$

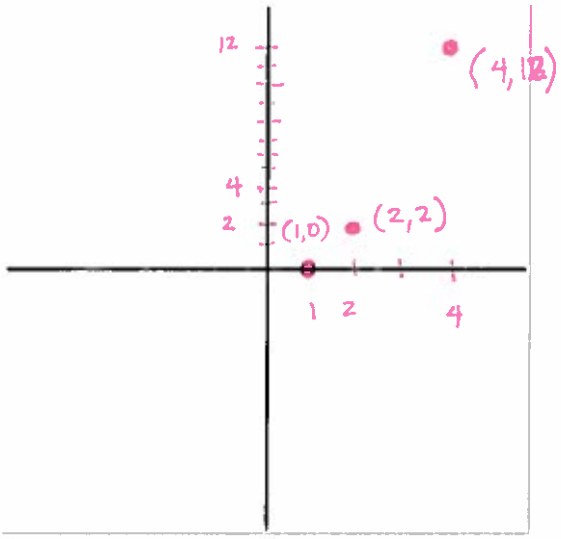
Range: $[-1, 8]$



32. (4 points) $h : \{1, 2, 4\} \to \mathbb{R}$ where $h(x) = x^2 - x$

Domain: $\{1, 2, 4\}$

Range: $\{0, 2, 12\}$



Sums and Sequences

For #1-3 decide whether the sequence is arithmetic, geometric, or neither. Write "arithmetic," "geometric," or "neither" on your answer sheet.

1. $45, 27, \frac{81}{5}, \ldots$ geometric: $r = \frac{3}{5}$
2. $-16, -10, -4, \ldots$ Arithmetic: $d = 6$
3. The sequence where $a_i = 3\left(\frac{2}{3}\right)^{i-1}$ $a_1 = 3$ $a_2 = 3 \cdot \frac{2}{3}$ $a_3 = 3\left(\frac{2}{3}\right)^2 \ldots$ geometric
4. Find the 201st term of the sequence: $8, 5, \frac{25}{8}, \ldots$

geometric, with $a_1 = 8$, $r = \frac{5}{8}$

$$a_{201} = a_1 \cdot r^{200} = 8 \cdot \left(\frac{5}{8}\right)^{200}$$

For #5-7, find the value of the sum or series.

5. $\sum_{i=1}^{\infty} \left(-5\left(-\frac{1}{4}\right)^{i-1}\right) = -5 + \frac{5}{4} - \frac{5}{16} + \ldots$ $a_1 = -5$ $r = \frac{-1}{4}$

$$= \frac{-5}{1 - \frac{-1}{4}} = \frac{-5}{5/4} = -4$$

6. $\sum_{i=1}^{50} (3i - 2)$ =

$= 1 + 4 + 7 + 10 + \ldots + 148$

Sum of 1st 50 terms of arith. sequence: $\frac{50(a_1 + a_{50})}{2} = \frac{50(149)}{2}$

7. $8 + 5 + \frac{25}{8} + \cdots$ = geometric series

$$= \frac{8}{1 - \frac{5}{8}} \qquad a_1 = 8 \qquad r = \frac{5}{8}$$

$$= \frac{8}{\frac{3}{8}} = \frac{64}{3}$$

Counting

For #8-10, your answer may include factorials (!) and binomial coefficients. You do not need to simplify these.

8. Youre on vacation and have brought with you 3 pair of shorts, 4 shirts and 2 pair of shoes. How many different outfits can you make?

$3 \cdot 4 \cdot 2 = 24$

options multiply

9. There are 100 sales people at a large corporation. The president of the corporation has decided she will select a head of sales, assistant to the head of sales, and secretary from the pool of 100. How many different options does the president have?

Choose 3 from 100. Order matters.

$$\frac{100!}{(100-3)!} = \frac{100!}{97!} = 100 \cdot 99 \cdot 98$$

(could also use options multiply)

10. If you have four different gifts and you want to give each one of your four best friends one of them, how many different ways can you give your gifts?

$4!$

(# of ways to order 4 friends)

(could also use options multiply)

11. Expand $(x - 2y)^5$. Your final answer should not include any factorials (!) or numbers that look like $\binom{n}{k}$.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$x^5 + 5x^4(-2y)^1 + 10x^3(-2y)^2 + 10x^2(-2y)^3 + 5x^1(-2y)^4 + x^0(-2y)^5$$

$$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$$

Functions

For #12-14, state the implied domain of each function using set or interval notation.

12. $f(x) = \dfrac{x^3 - 2x + 4}{x + 1}$

$x + 1 = 0 \Rightarrow x = -1$

$\mathbb{R} - \{-1\}$

13. $f(x) = \frac{x+1}{x}$

$\mathbb{R} - \{0\}$

14. $f(x) = \frac{1}{2}x^2 - 3x + 4$

$\mathbb{R}$

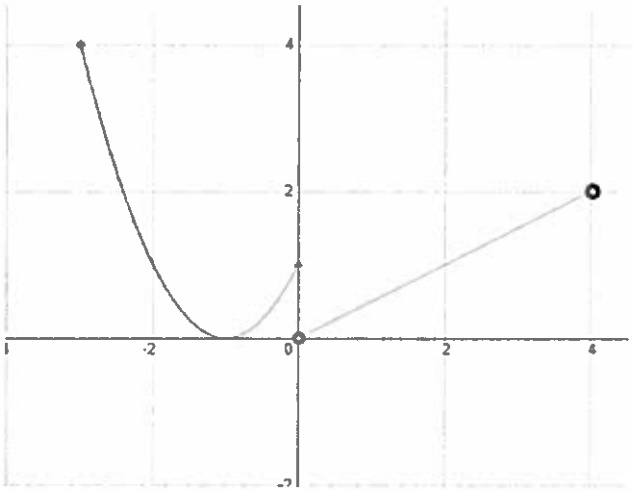
15. If $f(x) = x + 4$ and $g(x) = 2x$, find $f \circ g(2)$.

$f \circ g(2) = f(g(2)) = f(4) = 4 + 4 = 8$

16. If $f(x) = x^2 + 10$ and $g(x) = 10$, find $f \circ g(1)$.

$f \circ g(1) = f(g(1)) = f(10) = 10^2 + 10 = 110$

The graph of a function $f(x)$ is shown below. Use the graph to answer questions 15-20.



Note: The points $(0, 0)$ *and* $(4, 2)$ *are not in the graph. The points* $(-3, 4)$ *and* $(0, 1)$ *are in the graph.*

17. What is the y-intercept?

18. Find all x-intercepts.

19. What is the domain of $f(x)$?

20. What is the range of $f(x)$?

21. What are the intervals where $f(x) > 0$?

Inverse Functions For #22-23 Find the inverse of each function.

22. $k(x) = \dfrac{x}{x+1}$

$y = \dfrac{x}{x+1}$

$xy + y = x$

$xy - x = -y$

$x(y-1) = -y$

$x = \dfrac{-y}{y-1}$

23. $h(x) = 2x + 3$

$y = 2x + 3$

$y - 3 = 2x$

$\dfrac{y-3}{2} = x$

24. Decide whether the following functions are inverses. If they are, write "true." If not, write "false."

$$f : \mathbb{R} \to \mathbb{R} \text{ where } f(x) = \frac{x}{x+2}, \text{ and}$$

$$g : \mathbb{R} \to \mathbb{R} \text{ where } g(x) = \frac{x+2}{x}$$

$f \circ g(x) = f\left(\dfrac{x+2}{x}\right) = \dfrac{\left(\frac{x+2}{x}\right)}{\left(\frac{x+2}{x}\right)+2} = \dfrac{\frac{x+2}{x}}{\frac{x+2+2x}{x}}$

$= \dfrac{x+2}{x} \cdot \dfrac{x}{3x+2} = \dfrac{x+2}{3x+2}$

Graphing

For #25-32, state the domain and range of each function, and sketch the graph.

25. (3 points) $f(x) = -5$

26. (3 points) $g(x) = x^2$

27. (3 points) $h(x) = x$

28. (3 points) $F(x) = \frac{1}{x^2}$

29. (4 points) $G(x) = -\frac{1}{(x-2)^3}$

30. (4 points) $f(x) = (x-4)^3 + 5$

31. (4 points) $g : [-1, 2] \to \mathbb{R}$ where $g(x) = x^3$

32. (4 points) $h : \{1, 2, 4\} \to \mathbb{R}$ where $h(x) = x^2 - x$.