Homework 3 Example
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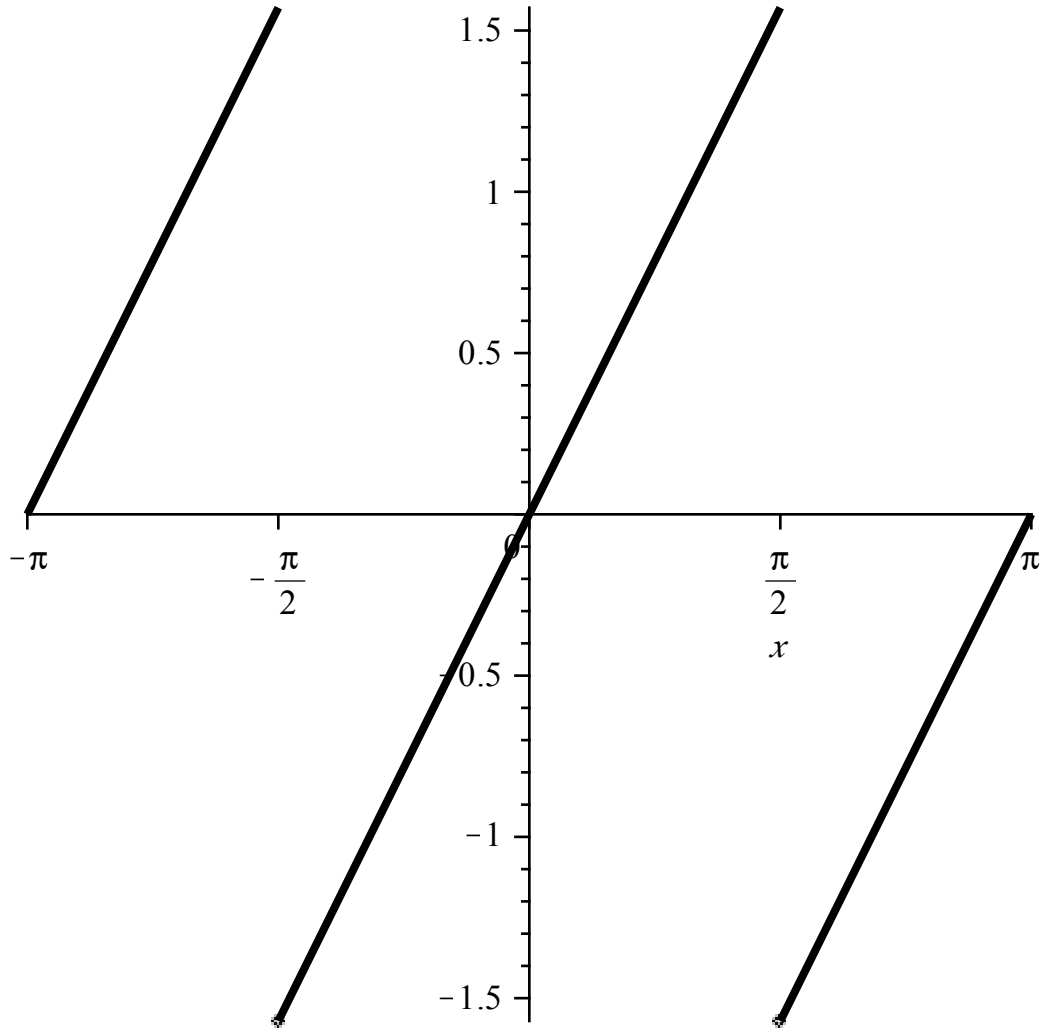
Suppose you were given the following graph in homework 3.

$$f := x \to piecewise\left(x < -\frac{\mathrm{Pi}}{2}, x + \mathrm{Pi}, x < \frac{\mathrm{Pi}}{2}, x, x < \frac{3\,\mathrm{Pi}}{2}, x - \mathrm{Pi}\right) : plot\left(\mathrm{f}(\mathrm{x}), \mathrm{x} = -\mathrm{Pi} .. \mathrm{Pi},\right.$$
$$\left. tickmarks = \left[spacing\left(\frac{\pi}{2}\right), default\right], color = black, thickness = 3, discont = true\right);$$



Then your solution would be as follows

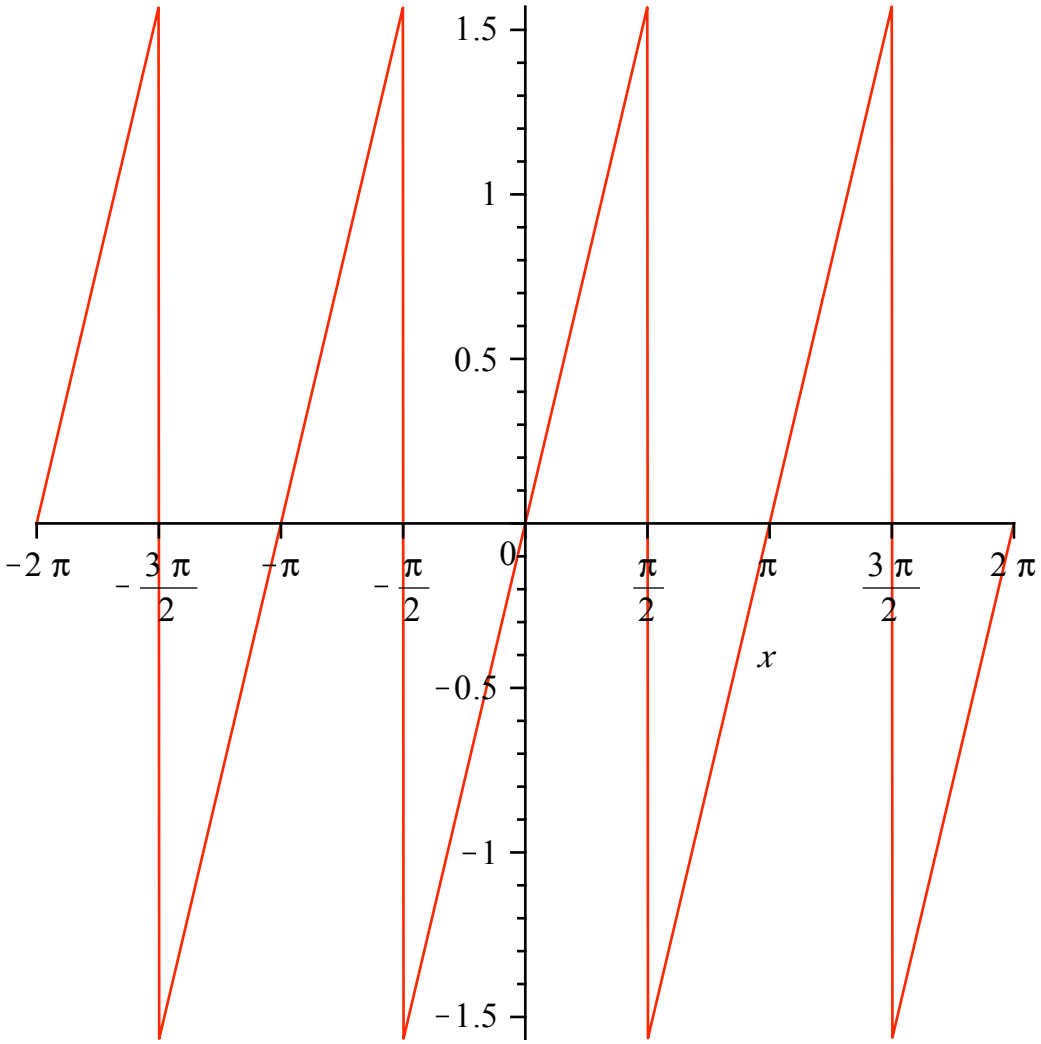
(a) This is the function for the sawtoth wave example from class on Sept. 2

$$sawtooth := x \to piecewise\left(x < -2\,\mathrm{Pi}, 0, x < -\frac{3\,\mathrm{Pi}}{2}, x + 2\,\mathrm{Pi}, x < -\frac{\mathrm{Pi}}{2}, x + \mathrm{Pi}, x < \frac{\mathrm{Pi}}{2}, x, x\right.$$
$$\left. < \frac{3\,\mathrm{Pi}}{2}, x - \mathrm{Pi}, x < 2\,\mathrm{Pi}, x - 2\,\mathrm{Pi}\right);$$

$$x \to piecewise\left(x < -2\,\pi, 0, x < -\frac{3}{2}\,\pi, x + 2\,\pi, x < -\frac{1}{2}\,\pi, x + \pi, x < \frac{1}{2}\,\pi, x, x < \frac{3}{2}\,\pi, x - \pi, x < 2\,\pi, x - 2\,\pi\right) \quad (1)$$

(b) Graph of the sawtooth wave

$$plot\left(sawtooth(x), x = -2\,\mathrm{Pi}..2\,\mathrm{Pi}, tickmarks = \left[spacing\left(\frac{\pi}{2}\right), default\right]\right)$$



(c) Compute the Fourier coefficients

a_0

$$\frac{1}{2\,\mathrm{Pi}}\,int(sawtooth(x), x = -\mathrm{Pi}..\mathrm{Pi});$$

$$0 \quad (2)$$

$$assume(n, integer, n > 0);$$

a_n

$$simplify\left(\frac{1}{\mathrm{Pi}}\,int(sawtooth(x)\cdot\cos(n\cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right);$$

$$0 \quad (3)$$

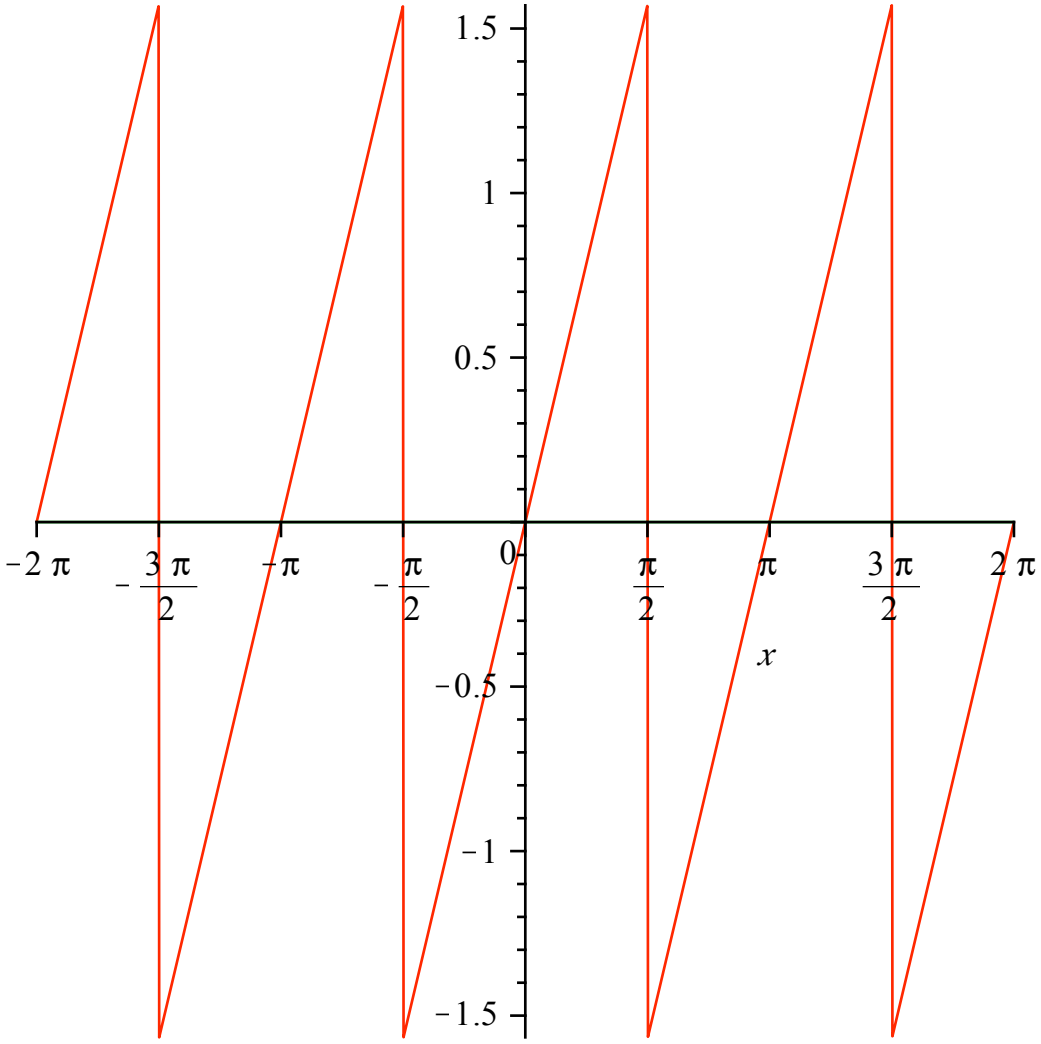
b_n

$$simplify\left(\frac{1}{\mathrm{Pi}}\ int(sawtooth(x)\cdot\sin(n\cdot x), x=-\mathrm{Pi}..\mathrm{Pi})\right);$$

$$-\frac{2\cos\left(\frac{1}{2}\,\pi\,n\sim\right)}{n\sim} \tag{4}$$

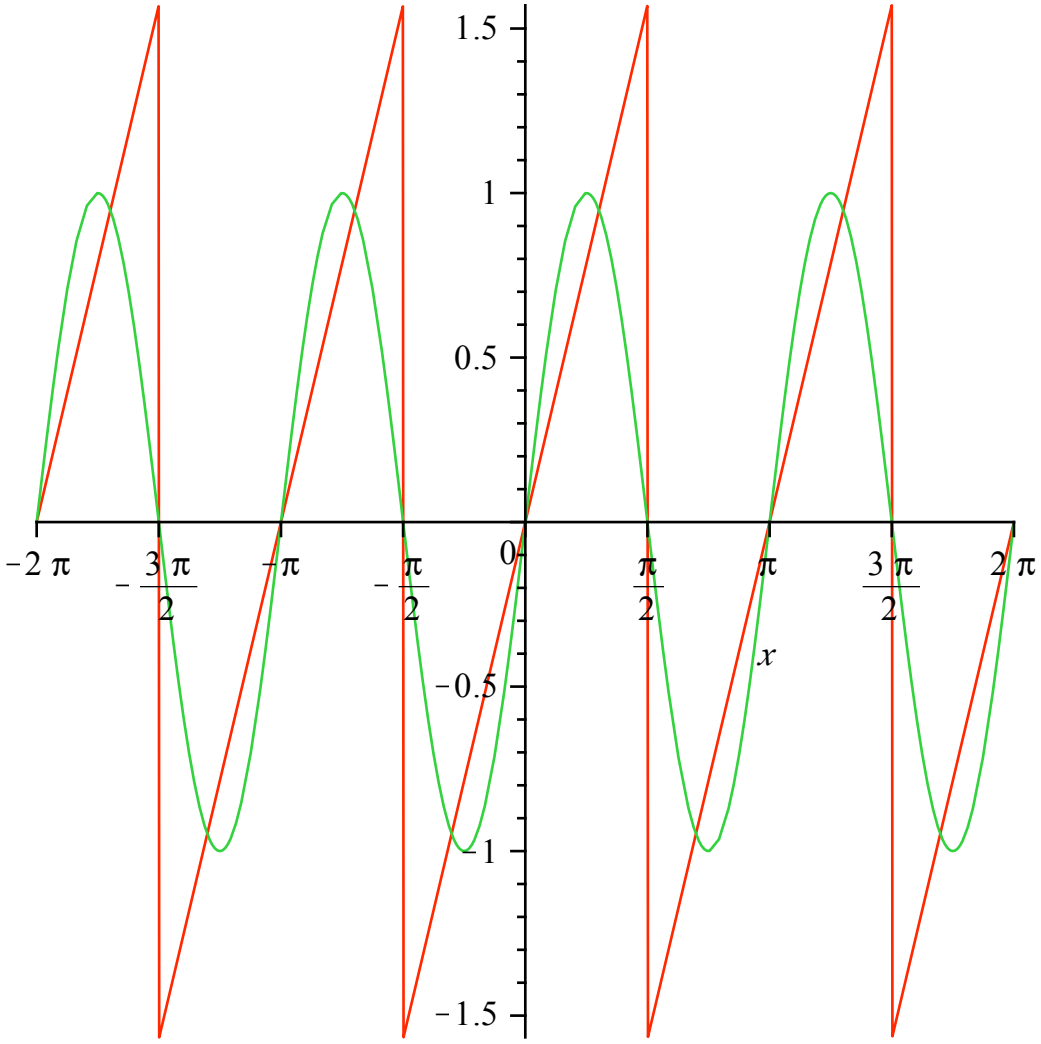
(d) graph of the function and S_1

$$plot\left(\left[sawtooth(x),\ \frac{1}{2\,\mathrm{Pi}}\ int(sawtooth(x), x=-\mathrm{Pi}..\mathrm{Pi}) + sum\left(\frac{1}{\mathrm{Pi}}\ int(sawtooth(x)\cdot\cos(m\cdot x), x=-\mathrm{Pi}..\mathrm{Pi})\cdot\cos(m\cdot x) + \frac{1}{\mathrm{Pi}}\ int(sawtooth(x)\cdot\sin(m\cdot x), x=-\mathrm{Pi}..\mathrm{Pi})\cdot\sin(m\cdot x),\ m=1..1\right)\right], x=-2\,\mathrm{Pi}..2\,\mathrm{Pi},\ tickmarks=\left[spacing\left(\frac{\pi}{2}\right), default\right]\right);$$



 # (e) graph of the function and S_2

$$plot\left(\left[sawtooth(x),\ \frac{1}{2\,\mathrm{Pi}}\ int(sawtooth(x), x=-\mathrm{Pi}..\mathrm{Pi}) + sum\left(\frac{1}{\mathrm{Pi}}\ int(sawtooth(x)\cdot\cos(m\cdot x), x=-\mathrm{Pi}..\mathrm{Pi})\cdot\cos(m\cdot x) + \frac{1}{\mathrm{Pi}}\ int(sawtooth(x)\cdot\sin(m\cdot x), x=-\mathrm{Pi}..\mathrm{Pi})\cdot\sin(m\cdot x),\ m=1..2\right)\right], x=-2\,\mathrm{Pi}..2\,\mathrm{Pi},\ tickmarks=\left[spacing\left(\frac{\pi}{2}\right), default\right]\right);$$



(f) graph of the function and S_5

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plot([sawtooth(x), 1/(2 Pi) int(sawtooth(x), x=-Pi..Pi) + sum(1/Pi int(sawtooth(x)·cos(m·x), x=
    -Pi..Pi)·cos(m·x) + 1/Pi int(sawtooth(x)·sin(m·x), x=-Pi..Pi)·sin(m·x), m=1..5)], x=
    -2 Pi..2 Pi, tickmarks=[spacing(π/2), default]);
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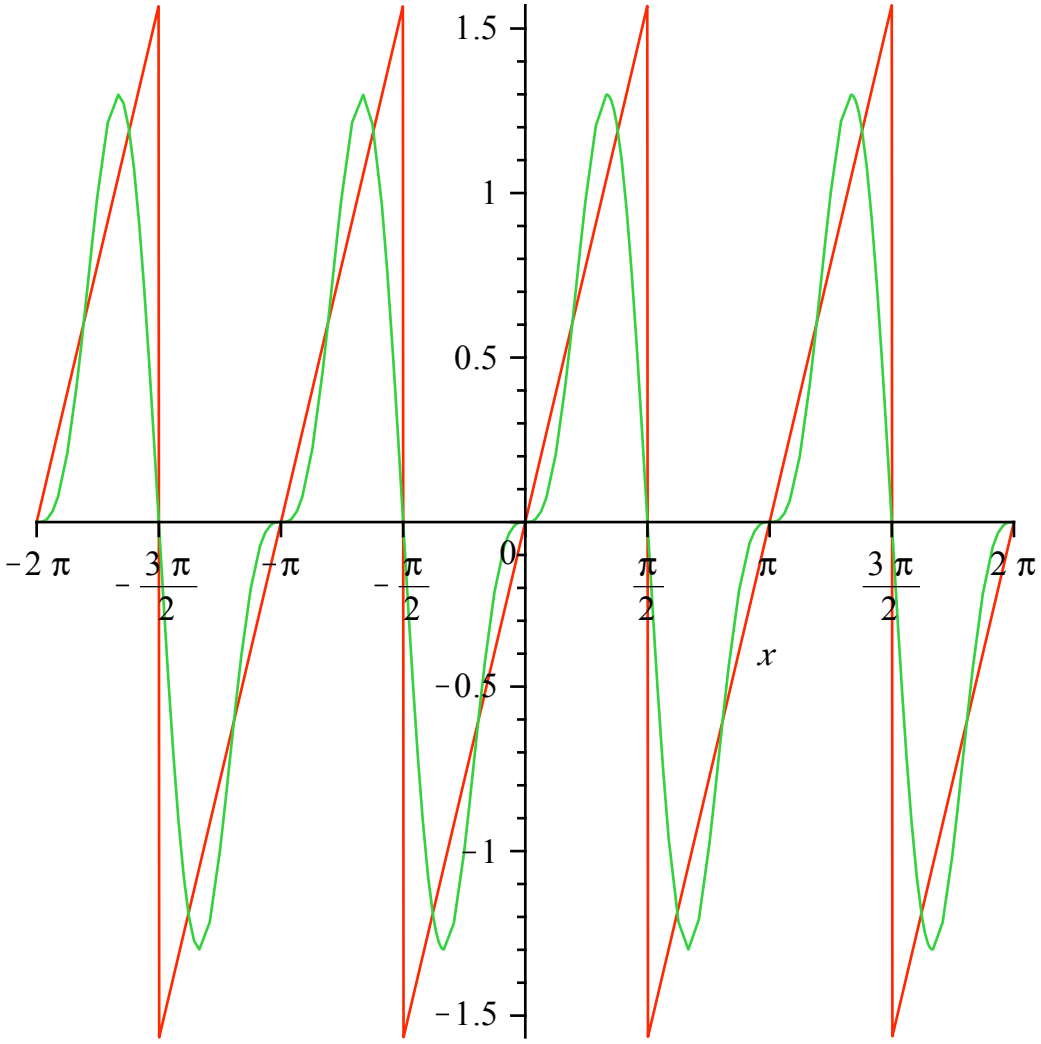

 # (g) graph of the function and S_{20}

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plot([[sawtooth(x), 1/(2 Pi) int(sawtooth(x), x=-Pi..Pi) + sum(1/Pi int(sawtooth(x)·cos(m·x), x=
    -Pi..Pi)·cos(m·x) + 1/Pi int(sawtooth(x)·sin(m·x), x=-Pi..Pi)·sin(m·x), m=1..20)], x=
    -2 Pi..2 Pi, tickmarks=[spacing(π/2), default]]);
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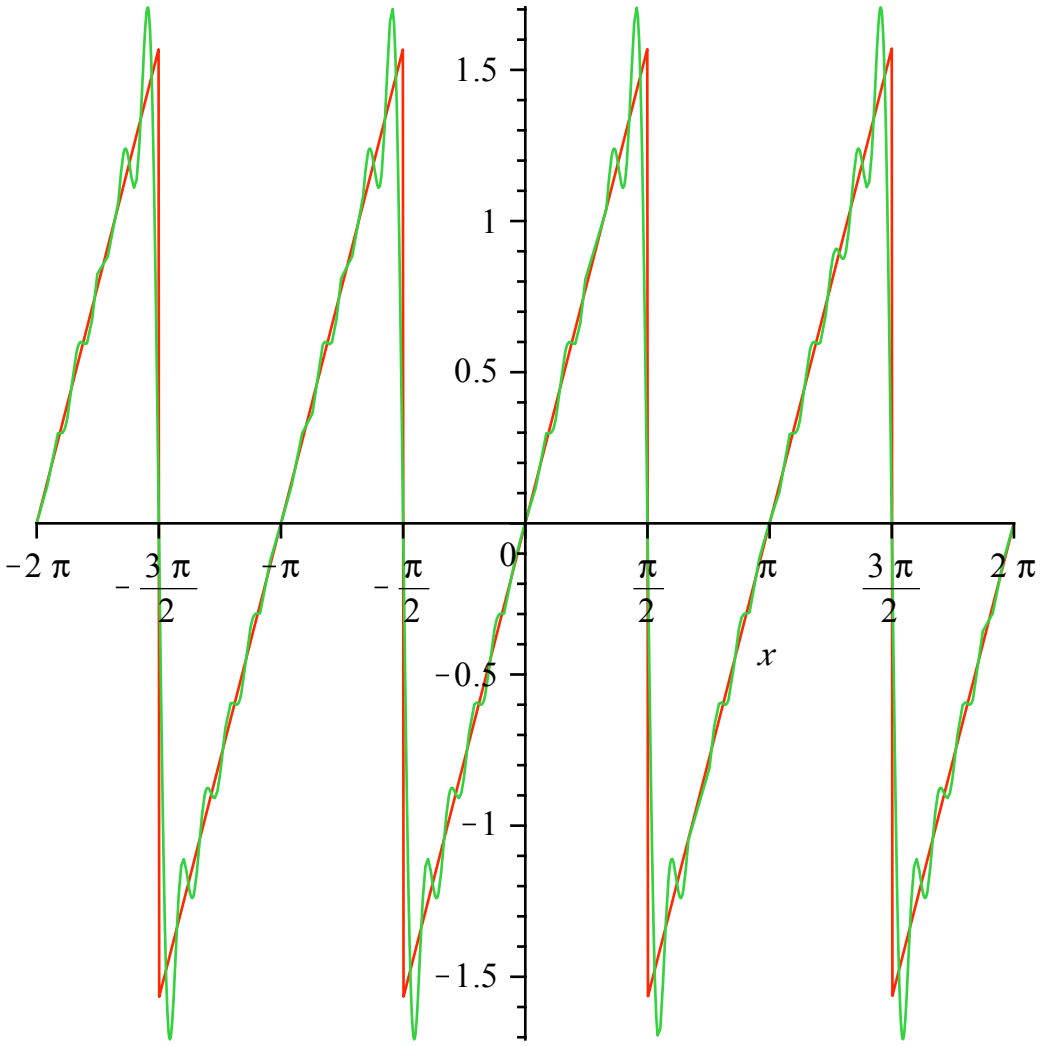
1.5
1
0.5
−2 π
−3 π/2
−π
−π/2
0
π/2
π
3 π/2
2 π
x
−0.5
−1
−1.5