MATH 1210-003 # Final Exam Spring 2011

NAME : _____

Sit in seat number:

You may not use a calculator. Your solutions must include enough justification that another person could understand and be convinced by your argument.
There are extra blank pages at the end of the booklet. If you need more room to work a problem please note the page number where your work continues.

QUESTION	VALUE	SCORE
1	15	
2	15	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	20	
11	15	
12	10	
13	15	
14	10	
15	80	
TOTAL	250	

1. (15 points)

$$f(x) = \begin{cases} 2x+1 & \text{for } x < 3 \\ 5 & \text{for } x = 3 \\ x^2 - 1 & \text{for } 3 < x \end{cases}$$

Find the following, or say why they do not exist:

$\lim_{x \to 3^+} f(x)$

$\lim_{x \to 3^-} f(x)$

$\lim_{x \to 3} f(x)$

$f(3)$

$\lim_{x \to 4^+} f(x)$

$\lim_{x \to 4^-} f(x)$

$\lim_{x \to 4} f(x)$

$f(4)$

Where is $f(x)$ continuous?

2. (15 points) Find all discontinuities of the following function and describe them (jump, vertical asymptote, removable discontinuity/hole).

$$\frac{(x+1)(x+2)(x+3)}{x(x+3)(x-3)}$$

3. (10 points) Compute the following three derivatives:

$(x^2+x+1)'$

$$\left(\frac{2x^2+x+1}{3x-5}\right)'$$

$$\frac{d}{dx}\sqrt{3x^2+4}$$

4. (10 points)

$$f(x) = x^3 - 3x^2 + 12 \qquad \text{for } S = [-2, 2]$$

Find the minimum and maximum value of f on S.

5. (10 points) Find the formula for the tangent line to $f(x) = 2x^3 - 10x + 5$ at $x = 2$.

6. (10 points) Find the solution to the differential equation

$$\frac{dy}{dx} = x$$

for which $y = 1$ when $x = 2$.

7. (10 points) Compute the following three integrals:

$$\int x + 1 \, dx$$

$$\int_{-2}^{3} 4x^2 + \frac{1}{x^2} \, dx$$

$$\int_{-\pi}^{\pi} \sin\theta \, d\theta$$

8. (10 points) Compute:

$$\int_0^3 t\sqrt{2t^2+1}\,dt$$

9. (10 points) Some values of the function f are given by the following table:

x	0	2	4	6
$f(x)$	2	4	1	5

Use a left Riemann sum to estimate

$$\int_0^6 f(x)\,dx$$

10. (20 points) Find the area of the region bounded by the curves $y = x + 2$ and $y = x^2$.

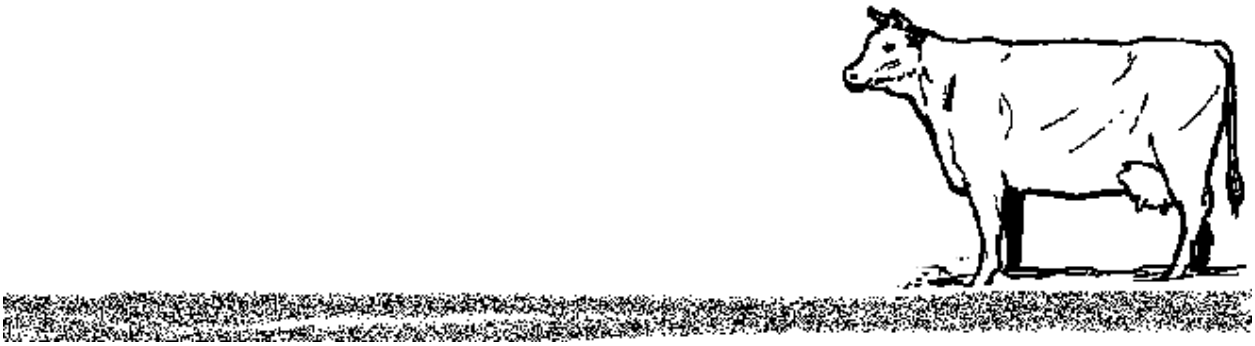
11. (15 points) Find the arc length of the parameterized curve:

$$\gamma(t) = (t^2 + 1,\, t^2 + 2) \qquad 1 \leq t \leq 4$$

12. (10 points) Gravel is being dumped from a conveyor belt at a rate of 20 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter and height are always the same. How fast is the height of the pile increasing when the pile is 18 feet high? Recall that the volume of a right circular cone with height h and radius of the base r is given by $V = \frac{\pi}{3}r^2h$.

13. (15 points) A cable that weighs 0.5 pounds per foot is used to lift 640 pounds of coal up a vertical mine shaft that is 900 feet deep. Find how much work was done.

14. (10 points) A cow is ready to head back to the barn for milking. The barn and the cow are at opposite corners of a rectangular pasture that measures 200ft by 300ft. There is a cow path along one of the long sides of the pasture. If the cow can walk 5 feet per second along the path and 3 feet per second across the pasture, what is the least amount of time it will take the cow to walk to the barn?



15. (80 points) (2-3 page Essay) Explain Calculus.

Hint: Here are some ideas to consider in your essay. What is the object of study in Calculus? Define, explain, and give examples of important terms like: limit, continuous, derivative, antiderivative, integral. Explain relationships between these concepts. For instance, is a continuous function always differentiable? Is a differentiable function always continuous? Explain why or give counterexamples.

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