MATH 3220 | Exam 2 | Spring 2009

NAME : ____________________

QUESTION	VALUE	SCORE
1	10	
2	15	
3	10	
4	10	
5	15	
6	20	
7	20	
8	15	
TOTAL	115	

1. (10 points) If $L\colon \mathbb{R}^p \to \mathbb{R}^q$ is a linear map, prove that L is differentiable.

2. (15 points) Find the best affine approximation near $(0,0)$ to the function $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$F(x, y) = (xy - 2x + y + 1\ ,\ x^2 + y^2 + x - 3y + 6)$$

3. (10 points) For the function $f(x,y) = x^2 + y^3 + 2xy$, find the direction of greatest ascent of f at $(1,1)$, and a direction in which the increase of the function is 0.

4. (10 points) Consider the function $F\colon \mathbb{R}^3 \to \mathbb{R}$ defined by:

$$F(x,y,z) = x^2 \sin y + 2x - z$$

The 0-level set of this function is a surface in $\mathbb{R}^3$. Give a smooth parameterization of this surface.

5. (15 points) Find the degree 2 Taylor approximation to the function $F\colon \mathbb{R}^2 \to \mathbb{R}$ defined by:

$$f(x, y) = \ln(x + xy)$$

near the point $(1, 0)$.

6. (20 points) Consider the function $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$F(r,t) = (r\cos t\ ,\ r\sin t)$$

Apply the Inverse Function Theorem to determine points in the domain that have a smooth local inverse. The function is not one to one. Consider the point $(1,0)$ in the range. Find two distinct local inverses to F defined on a neighborhood of $(1,0)$.

7. (20 points) For the system of equations

$$x^2 + y^2 - z^2 = 0$$
$$x + y + z = 0$$

at which points of the solution set S is there a neighborhood in which S is a smooth curve? At each such point, find an equation of the tangent line.

8. (15 points) Consider the function $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$F(x, y, z) = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

For each point in the domain, determine whether or not F has a local inverse.