MATH 3220 — Exam 1 — Spring 2009

Exam 1

NAME : ______________________________

QUESTION	VALUE	SCORE
1	10	
2	10	
3	15	
4	30	
5	15	
6	30	
TOTAL	110	

1. (10 points) Is the set $\{(x,0) \in \mathbb{R}^2 \mid -1 < x < 1\}$ open, closed, both, or neither in $\mathbb{R}^2$?

2. (10 points) Is the following set connected in $\mathbb{R}^2$:

$$\{(x,y) \mid x^2 + y^2 < 1\} \cup \{(x,1) \mid x \in \mathbb{R}\}$$

3. (15 points)

(a) Define "convergent sequence".

(b) Does the sequence $(x_n)_{n\in\mathbb{N}}$,

$$x_n = \left(1 + \frac{(-1)^n}{n}, \sin\left(\frac{1}{n}\right)\right) \in \mathbb{R}^2$$

converge? If so, to what?

4. (30 points) Choose one of the following statements to prove. Indicate which one you choose.

(a) Let A be a set in a topological space X. A is closed if and only if every sequence in A that converges in X converges to a point in A.

(b) Let A be a set in a metric space X. A point $a \in A$ is an interior point of A if and only if for any sequence (x_n) in X converging to a there is some N such that for all $n > N$, $x_n \in A$.

(c) Let A be a set in a topological space X. A is disconnected if and only if there is a continuous map $f\colon A \to \mathbb{R}$ whose image is the set $\{0, 1\}$.

5. (15 points) Does the sequence of functions (F_n) converge on $B_1\mathbf{0} \subset \mathbb{R}^3$, where $F_n\colon \mathbb{R}^3 \to \mathbb{R}^3$ is defined by:

$$F(x, y, z) = \left(\sin\left(\frac{xy}{n}\right), \frac{1}{n}, x^n + y^n + z^n\right)$$

If so, is the convergence uniform?

6. (30 points)

(a) State the Bolzano-Weierstrass Theorem.

(b) Prove the Bolzano-Weierstrass Theorem for $\mathbb{R}^3$. You may assume the theorem is true for $\mathbb{R}$.