$assume(m, integer, m > 0) : assume(n, integer, n > 0) :$

#1 2d wave equation with initial position f(x,y)=1

$c := 1 :$

$a := 1 :$

$b := 1 :$

$f := (x, y) \to 1 :$

$\lambda := (m, n) \to c \cdot \mathrm{Pi} \cdot \mathrm{sqrt}\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right); \mathrm{phi} := (m, n, x, y) \to \sin\left(\frac{m \cdot \mathrm{Pi} \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot y}{b}\right);$

$$(m, n) \to c\,\pi \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$(m, n, x, y) \to \sin\left(\frac{m\,\pi\,x}{a}\right) \sin\left(\frac{n\,\pi\,y}{b}\right) \quad \textbf{(1)}$$

$B := (m, n) \to \frac{4}{a \cdot b} int(int(f(x, y) \cdot \mathrm{phi}(m, n, x, y), x = 0 ..a), y = 0 ..b);$

$$(m, n) \to \frac{4 \left(\int_0^b \int_0^a f(x, y)\, \phi(m, n, x, y)\, \mathrm{d}x\, \mathrm{d}y\right)}{a\, b} \quad \textbf{(2)}$$

 # The following function is the sum for m from 1 to p and for n from 1 to q of $u_{m, n}(x, y, t)$

$u := (p, q, x, y, t) \to (add(add(B(m, n) \cdot \cos(\lambda(m, n) \cdot t) \cdot \mathrm{phi}(m, n, x, y), m = 1 ..\mathrm{p}), n = 1 ..\mathrm{q}))$

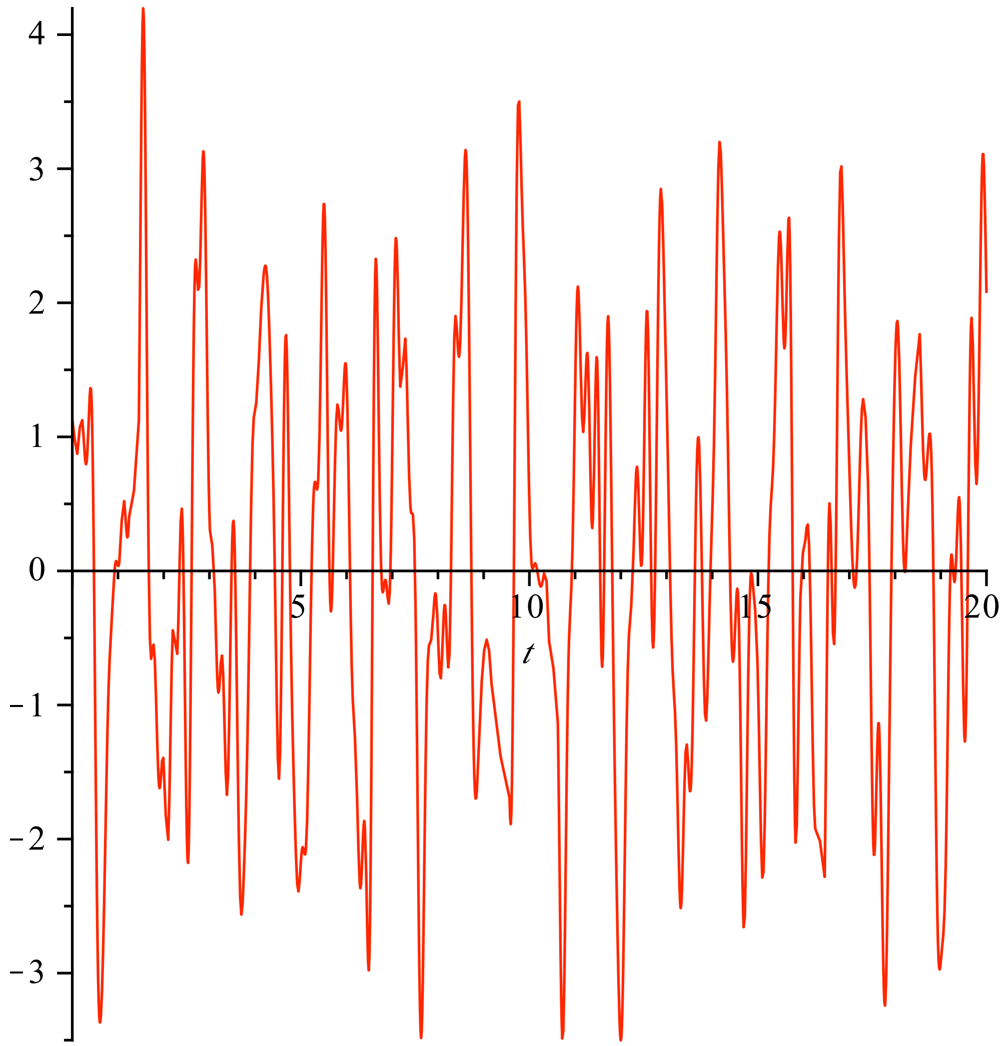
$$(p, q, x, y, t) \to add(add(B(m, n) \cos(\lambda(m, n)\, t)\, \phi(m, n, x, y), m = 1 ..p), n = 1 ..q) \quad \textbf{(3)}$$

$\#plots[animate](plot3d, [u(10, 10, x, y, t), x = 0 ..a, y = 0 ..b], t = 0 ..200, frames = 100)$

 # depending on your computer this animation may take a while to compute.

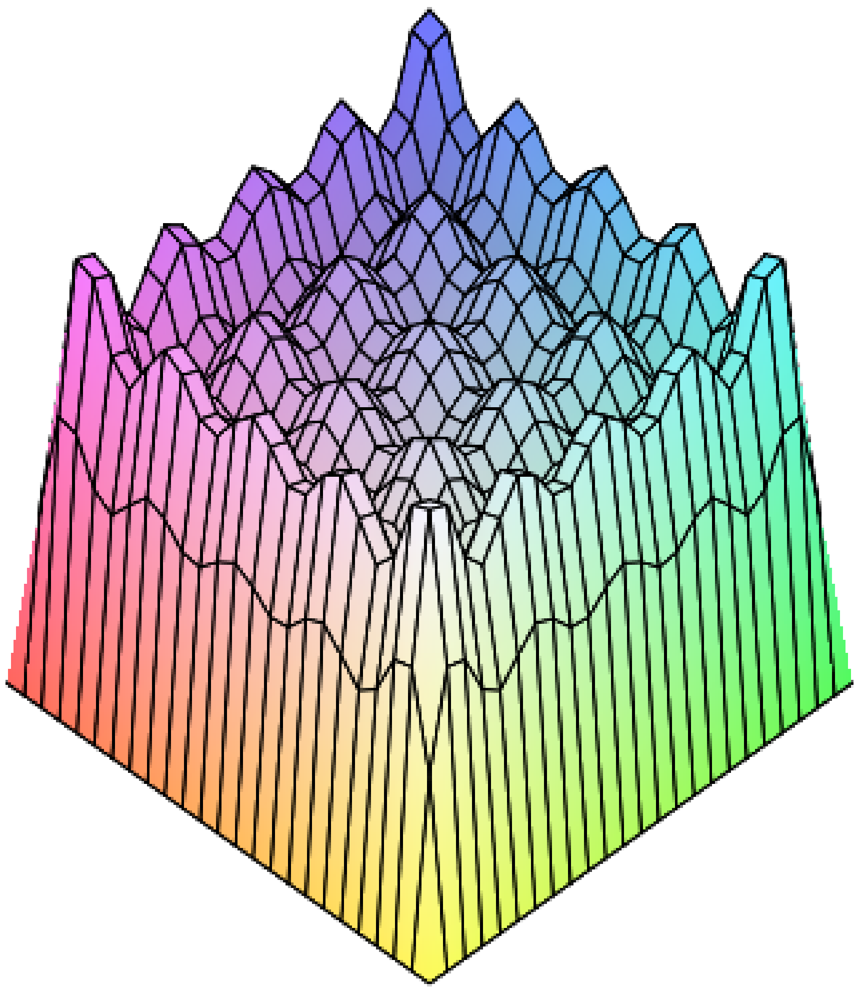
$plot(u(10, 10, .5, .5, t), t = 0 ..20);$

 # here is a plot of the point at the center of the membrane (x,y)=(.5,.5) as time varies from 0 to 20. Note it is not periodic.

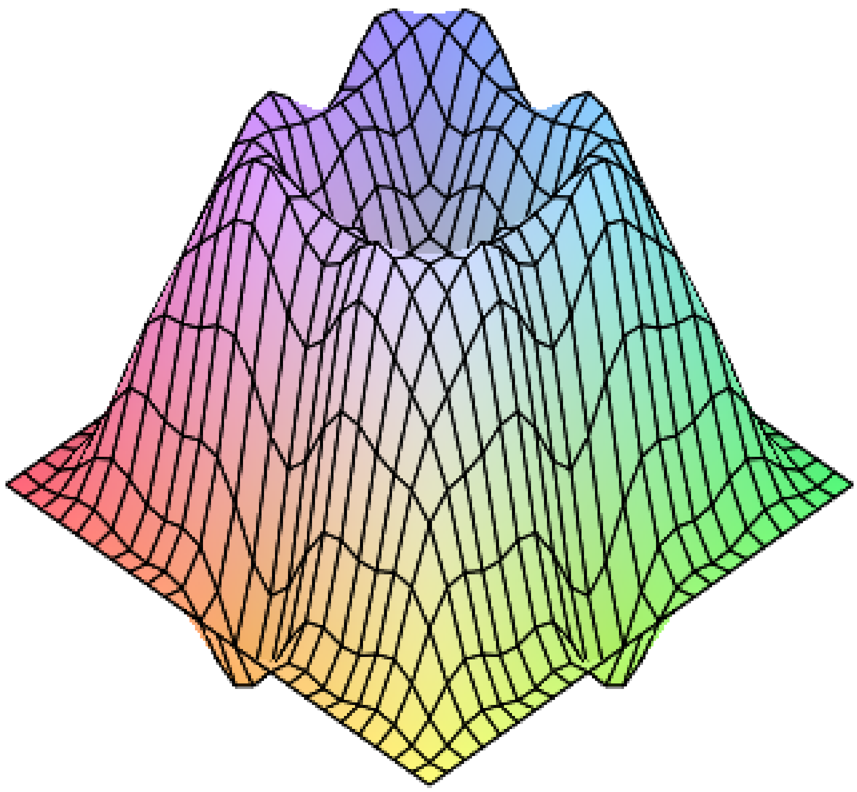


#Here are some plots of the shape of the membrane at fixed time t=0 and t=10.

$plot3d\,(u(10, 10, x, y, 0), x = 0..a, y = 0..b)$



plot3d (*u* (10, 10, *x*, *y*, 10), *x* = 0 ..*a*, *y* = 0 ..*b*)



#2 2 d wave equation. This time the membrane is large and the inital position is 1 near the center of the plate and 0 elsewhere.

$c := 1:$

$a := 100:$

$b := 100:$

$f := (x, y) \rightarrow piecewise(x < 50.1 \textbf{ and } x > 49.9 \textbf{ and } y < 50.1 \textbf{ and } y > 49.9, 1, 0):$

$\lambda := (m, n) \rightarrow c \cdot \mathrm{Pi} \cdot \mathrm{sqrt}\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right)$; $\mathrm{phi} := (m, n, x, y) \rightarrow \sin\left(\frac{m \cdot \mathrm{Pi} \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot y}{b}\right)$;

$$(m, n) \rightarrow c\,\pi\,\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$(m, n, x, y) \rightarrow \sin\left(\frac{m \pi x}{a}\right) \sin\left(\frac{n \pi y}{b}\right) \quad \textbf{(4)}$$

$$B := (m, n) \rightarrow \frac{4}{a \cdot b} int\left(int\left(f(x, y) \cdot \text{phi}(m, n, x, y), x = 0 ..a\right), y = 0 ..b\right)$$

$$(m, n) \rightarrow \frac{4 \left(\int_0^b \int_0^a f(x, y)\, \phi(m, n, x, y)\, dx\, dy\right)}{a\, b} \quad \textbf{(5)}$$

 # The following function is the sum for m from 1 to p and for n from 1 to q of $u_{m,\,n}(x, y, t)$

$$u := (p, q, x, y, t) \rightarrow \left(add\left(add\left(B(m, n) \cdot \cos(\lambda(m, n) \cdot t) \cdot \text{phi}(m, n, x, y), m = 1 ..\text{p}\right), n = 1 ..\text{q}\right)\right)$$

$$(p, q, x, y, t) \rightarrow add\left(add\left(B(m, n) \cos(\lambda(m, n)\, t)\, \phi(m, n, x, y), m = 1 ..p\right), n = 1 ..q\right) \quad \textbf{(6)}$$

$$\#plots[animate]\left(plot3d, \left[u(10, 10, x, y, t), x = 0 ..a, y = 0 ..b\right], t = 0 ..200, frames = 50\right)$$

#3 This is a heat equation with initial temperature a constant 1 and boundaries at 0

$$c := \frac{1}{10} :$$

$$a := 1 :$$

$$b := 1 :$$

$$f := (x, y) \rightarrow 1 :$$

$$\lambda := (m, n) \rightarrow \text{sqrt}\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right) : \text{phi} := (m, n, x, y) \rightarrow \sin\left(\frac{m \cdot \text{Pi} \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot y}{b}\right) :$$

$$B := (m, n) \rightarrow \frac{4}{a \cdot b} int\left(int\left(f(x, y) \cdot \text{phi}(m, n, x, y), x = 0 ..a\right), y = 0 ..b\right)$$

$$(m, n) \rightarrow \frac{4 \left(\int_0^b \int_0^a f(x, y)\, \phi(m, n, x, y)\, dx\, dy\right)}{a\, b} \quad \textbf{(7)}$$

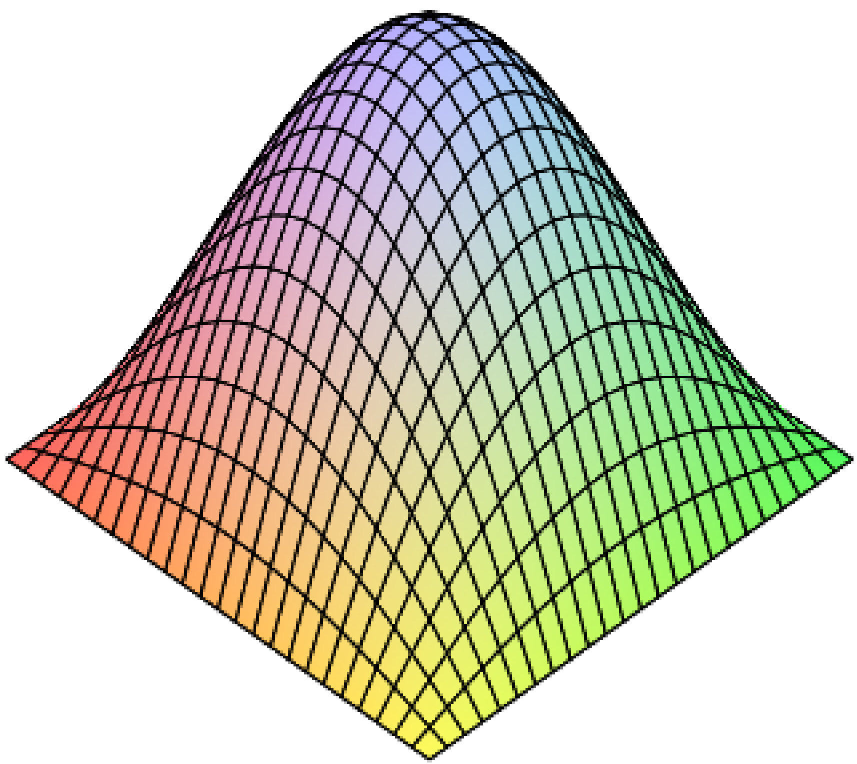
$$u := (p, q, x, y, t) \rightarrow \left(add\left(add\left(B(m, n) \cdot \exp\left(-\pi^2 \cdot c^2 \cdot (\lambda(m, n))^2 \cdot t\right) \cdot \text{phi}(m, n, x, y), m = 1 ..\text{p}\right), n = 1 ..\text{q}\right)\right)$$

$$(p, q, x, y, t) \rightarrow add\left(add\left(B(m, n)\, e^{-\pi^2 c^2 \lambda(m, n)^2 t}\, \phi(m, n, x, y), m = 1 ..p\right), n = 1 ..q\right) \quad \textbf{(8)}$$

$$\#plots[animate]\left(plot3d, \left[u(10, 10, x, y, t), x = 0 ..a, y = 0 ..b\right], t = 0 ..10, frames = 40\right)$$

 # Here's a plot for time t=5. This one is also a better approximation, p=q=20 so there are more terms of the series included.

$plot3d\,(u(20, 20, x, y, 5), x = 0\,..a, y = 0\,..b)$



#4 This one is a steady state problem when one of the boundary edges is fixed at temp 100 and the others are 0.

$C := \frac{1}{10}$:
$A := 1$:
$B := 1$:
$F_2 := x \rightarrow 100$:

$assume(M, integer, M > 0)$: $assume(N, integer, N > 0)$:
$\text{Lambda} := (M, N) \rightarrow \text{sqrt}\left(\left(\frac{M}{A}\right)^2 + \left(\frac{N}{B}\right)^2\right)$; $\text{Phi} := (M, N, X, Y) \rightarrow \sin\left(\frac{M \cdot \text{Pi} \cdot X}{A}\right) \cdot \sin\left(\frac{N \cdot \text{Pi} \cdot Y}{B}\right)$;

$$(M, N) \rightarrow \sqrt{\frac{M^2}{A^2} + \frac{N^2}{B^2}}$$

$$(M, N, X, Y) \rightarrow \sin\left(\frac{\pi M X}{A}\right) \sin\left(\frac{N \pi Y}{B}\right) \tag{9}$$

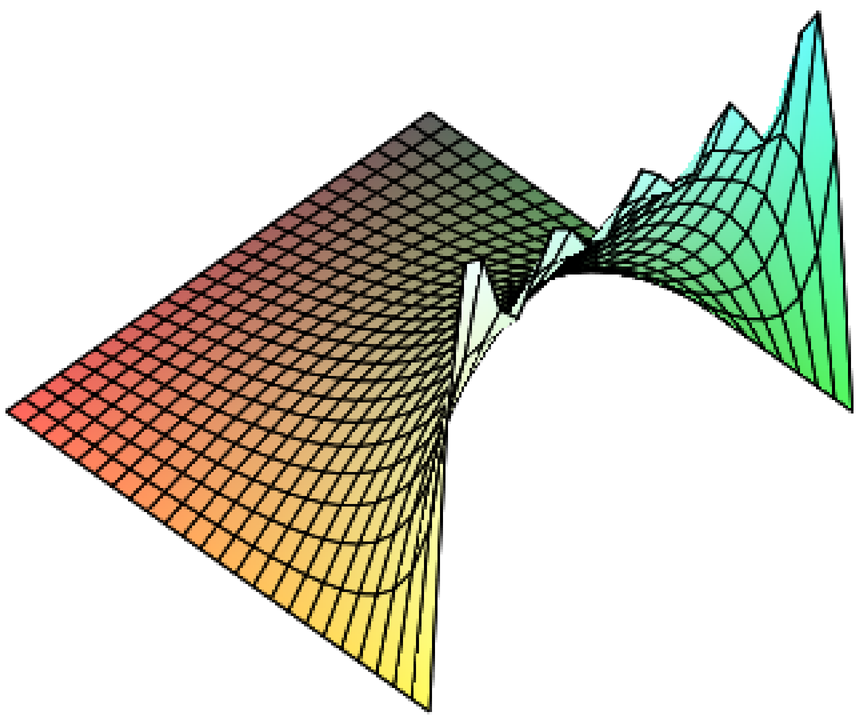
$Bsscoefficient := (N) \rightarrow \dfrac{2}{\sinh(B \cdot N \cdot \mathrm{Pi})} int\left(F_2(X) \cdot \sin\left(\dfrac{N \cdot \mathrm{Pi} \cdot X}{A}\right), X = 0 ..A\right);$

$$N \rightarrow \frac{2 \left(\int_0^A F_2(X) \sin\left(\frac{\pi N X}{A}\right) \mathrm{d}X\right)}{\sinh(B N \pi)} \tag{10}$$

$Uss := (P, X, Y) \rightarrow add\left(Bsscoefficient(R) \cdot \sin\left(\dfrac{\mathrm{R \cdot Pi \cdot X}}{A}\right) \cdot \sinh\left(\dfrac{\mathrm{R \cdot Pi \cdot Y}}{B}\right), \mathrm{R} = 1 ..\mathrm{P}\right)$

$$(P, X, Y) \rightarrow add\left(Bsscoefficient(R) \sin\left(\frac{R \pi X}{A}\right) \sinh\left(\frac{R \pi Y}{B}\right), R = 1 ..P\right) \tag{11}$$

$plot3d\,(Uss(10, X, Y), X = 0 ..A, Y = 0 ..B)$



#5 This heat problem combines the previous 2. Inital temp is constant 100 with one of the boundary sides fixed at 100 and the others fixed at 0

$c := \frac{1}{10}:$

$a := 1:$

$b := 1:$

$f := (x, y) \to 100:$

$\lambda := (m, n) \to \mathrm{sqrt}\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right); \mathrm{phi} := (m, n, x, y) \to \sin\left(\frac{m \cdot \mathrm{Pi} \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot y}{b}\right);$

$$(m, n) \to \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$(m, n, x, y) \to \sin\left(\frac{m \pi x}{a}\right) \sin\left(\frac{n \pi y}{b}\right) \quad \textbf{(12)}$$

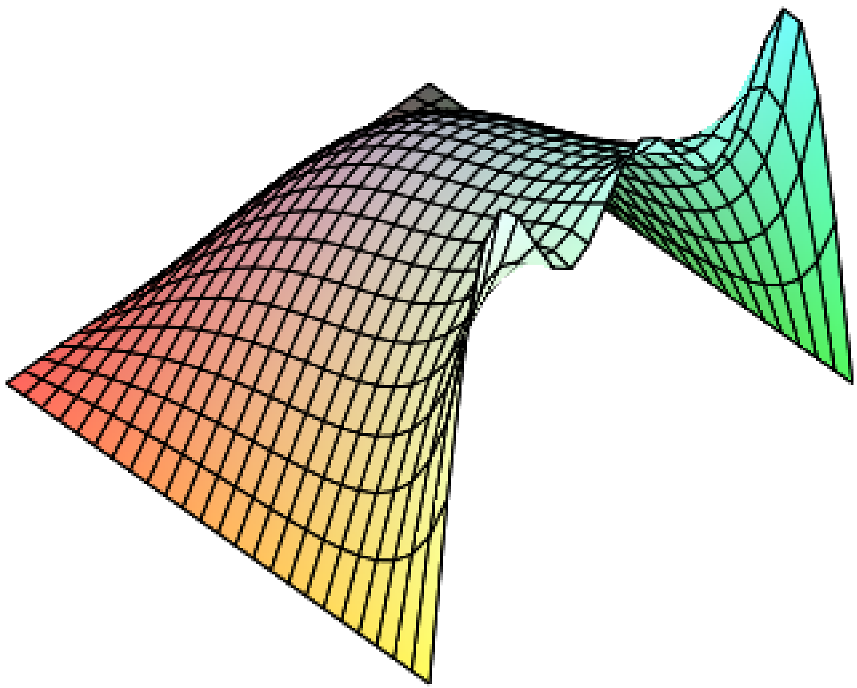
$Bcoefficient := (m, n) \to \frac{4}{a \cdot b} int\left(int\left((f(x, y) - Uss(5, x, y)) \cdot \mathrm{phi}(m, n, x, y), x = 0..a\right), y = 0..b\right)$

$$(m, n) \to \frac{4 \left(\int_0^b \int_0^a (f(x, y) - Uss(5, x, y))\, \phi(m, n, x, y)\, dx\, dy\right)}{a\, b} \quad \textbf{(13)}$$

$u := (p, q, x, y, t) \to \left(add\left(add\left(Bcoefficient(m, n) \cdot \exp\left(-\pi^2 \cdot c^2 \cdot \left(\lambda(m, n)\right)^2 \cdot t\right) \cdot \mathrm{phi}(m, n, x, y), m = 1..\mathrm{p}\right), n = 1..\mathrm{q}\right)\right) + Uss(5, x, y)$

$$(p, q, x, y, t) \to add\left(add\left(Bcoefficient(m, n)\, e^{-\pi^2 c^2 \lambda(m, n)^2 t}\, \phi(m, n, x, y), m = 1..p\right), n = 1..q\right) + Uss(5, x, y) \quad \textbf{(14)}$$

$plot3d(u(5, 5, x, y, 5), x = 0..a, y = 0..b);$



$\#plots[animate](plot3d, [u(10, 10, x, y, t), x = 0..a, y = 0..b], t = 0..10, frames = 40)$