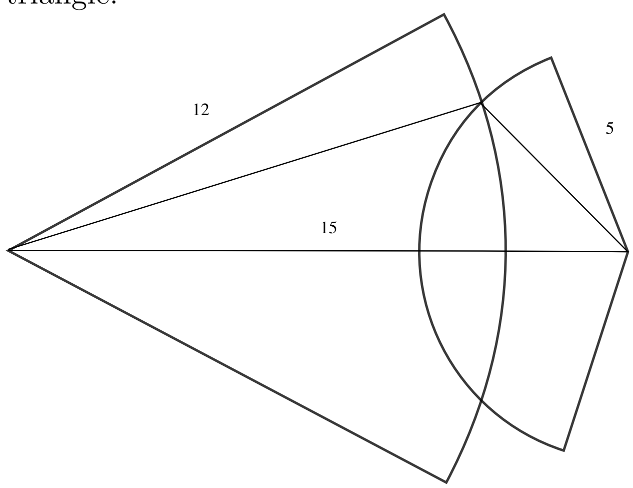
MATH 1060 Quiz 8 Fall 2008

Solutions

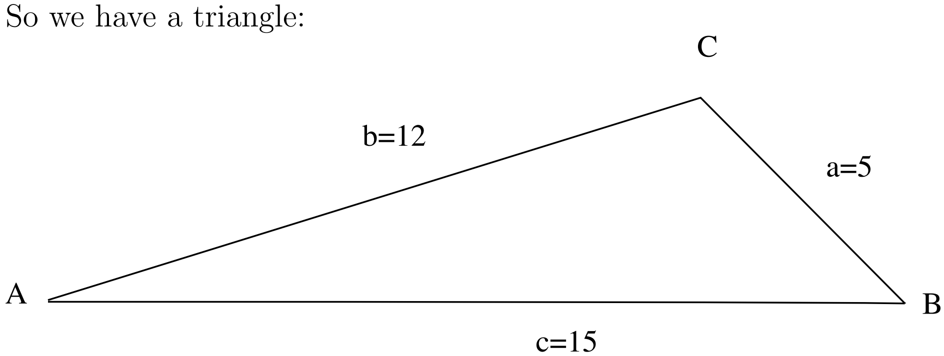
1. (5 points) Decide whether or not there is a triangle with side lengths $a = 5$, $b = 12$, and $c = 15$. If so, find its three angles.

Solution:

Yes, there is a triangle with these three side lengths, since c is the longest side and $a+b > c$. You could show this with a sketch by drawing a segment of length 15 for he longest side. Then draw circles centered at each edge of the segment with radii equal to a and b and see that the two circles intersect. The point where they intersect is the third vertex of the triangle:



So we have a triangle:



One method would be to use the Law of Cosines three times to find the three angles. To find angle C:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

so

$$C = \arccos(\frac{c^2 - a^2 - b^2}{-2ab}) = \arccos(\frac{56}{-120}) \approx 2.06 \approx 118.03^\circ$$

Similarly:

$$A = \arccos(\frac{a^2 - b^2 - c^2}{-2bc}) \approx .30 \approx 17.19^\circ$$

$$B = \arccos(\frac{b^2 - a^2 - c^2}{-2ac}) \approx .79 \approx 45.26^\circ$$

You could have also used the Law of Cosines twice and then subtracted the two agles form π to get the third.

Another method would be to use the Law of Cosines once and then use the Law of Sines twice to find the other two angles, or use the Law of Sines once and then subtract the two angles from π to get the third.

When using the Law of Sines you must be careful to consider whether you need arcsin or $\pi -$ arcsin. You can avoid this problem by always doing the Law of Cosines in the first step to find the largest angle. Then the other two angles must be acute, so you only need to take arcsin in you solutions.

For example, if you found C with the Law of Cosines then you could ust the Law of Sines to find:

$$B = \arcsin(\frac{b \sin C}{c})$$

and

$$A = \arcsin(\frac{a \sin C}{c})$$

But if you found A with the Law of Cosines then you need:

$$B = \arcsin(\frac{b \sin A}{a})$$

and

$$C = \pi - \arcsin(\frac{c \sin A}{a})$$

2. (5 points) Decide whether or not there is a triangle with side lengths $a = 6$, $b = 4$, and $c = 5$. If so, find its three angles.

Solution:

Yes, there is a triangle, since the sum of the shorter two sides is longer than the longest side. This is a SSS, just like problem 1, so you can use the same method of solution.

One method would be to use the Law of Cosines three times to find the three angles. The longest side is a, so first find angle A using the Law of Cosines.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

so

$$A = \arccos(\frac{a^2 - b^2 - c^2}{-2bc}) \approx 1.45 \approx 82.82^\circ$$

Angle A is the largest angle, since it is opposite the largest side, and it is acute, so all the angles are acute. If you choose to use the Law of Sines to find angles B and C then you should pick the arcsin solution in each case, because that is the choice that gives an acute angle.
Or, jsut use the Law of Cosines two more times, and find:

$$B = \arccos(\frac{b^2 - a^2 - c^2}{-2ac}) \approx .72 \approx 41.41^\circ$$

$$C = \arccos(\frac{c^2 - a^2 - b^2}{-2ab}) \approx .97 \approx 55.77^\circ$$