Solutions

1. Find all x that satisfy the equation

$$5\tan(2x-1)+1=4$$

Solution:

$$\begin{aligned}
5\tan(2x-1)+1&=4\\
5\tan(2x-1)&=3\\
\tan(2x-1)&=\frac{3}{5}\\
2x-1&=\arctan(\frac{3}{5})+\pi n\\
2x&=1+\arctan(\frac{3}{5})+\pi n\\
x&=\frac{1}{2}+\frac{1}{2}\arctan(\frac{3}{5})+\frac{\pi}{2}n
\end{aligned}$$

2. Find all solutions to

$$\frac{1}{2}\sin^2(2x)-\frac{5}{2}\sin(2x)=-\frac{3}{2}+\frac{1}{2}\cos^2(2x)$$

Solution:

$$\begin{aligned}
\frac{1}{2}\sin^2(2x)-\frac{5}{2}\sin(2x)&=-\frac{3}{2}+\frac{1}{2}\cos^2(2x)\\
\frac{1}{2}\sin^2(2x)-\frac{5}{2}\sin(2x)&=-\frac{3}{2}+\frac{1}{2}(1-\sin^2(2x))
\end{aligned}$$

substitute $z = \sin(2x)$

$$\begin{aligned}
\frac{1}{2}z^2 - \frac{5}{2}z &= -\frac{3}{2} + \frac{1}{2}(1 - z^2) \\
\frac{1}{2}z^2 - \frac{5}{2}z &= -\frac{3}{2} + \frac{1}{2} - \frac{1}{2}z^2 \\
z^2 - \frac{5}{2}z &= -1 \\
z^2 - \frac{5}{2}z + 1 &= 0 \\
z &= \frac{\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}}{2} \\
z &= \frac{\frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}}}{2} \\
z &= \frac{\frac{5}{2} \pm \sqrt{\frac{9}{4}}}{2} \\
z &= \frac{\frac{5}{2} \pm \frac{3}{2}}{2} \\
z &= \frac{5}{4} \pm \frac{3}{4} \\
z &= 2 \text{ or } \frac{1}{2}
\end{aligned}$$

$z = \sin(2x)$, so we have

$$\sin(2x) = 2 \text{ or } \sin(2x) = \frac{1}{2}$$

But we can't have $\sin(2x) = 2$, since the range of sin is $[-1, 1]$, so the only possibility is $\sin(2x) = \frac{1}{2}$.

$$\begin{aligned}
\sin(2x) &= \frac{1}{2} \\
2x &= \arcsin(\frac{1}{2}) + 2\pi n && \text{or } 2x = \pi - \arcsin(\frac{1}{2}) + 2\pi n \\
x &= \frac{1}{2}\arcsin(\frac{1}{2}) + \pi n && \text{or } x = \frac{\pi}{2} - \frac{1}{2}\arcsin(\frac{1}{2}) + \pi n
\end{aligned}$$

We know that $\arcsin(\frac{1}{2}) = \frac{\pi}{6}$, so

$$x = \frac{\pi}{12} + \pi n \text{ or } x = \frac{5\pi}{12} + \pi n$$