MATH 1060 # Exam 1, version B Fall 2008

Solutions

1. (5 points)

(a) Is the following true or false? $(a+b)^2 = a^2 + b^2$

(b) What is the value of the function $f(x) = 2x^2 + 3$ at 3?

Solution:

(a) False. If $a = b = 1$ then $(a+b)^2 = 4$ but $a^2 + b^2 = 2$.

(b) $f(3) = 21$

2. (5 points) Find the midpoint of the line segment joining $(0, 0)$ to $(3, -2)$.

Solution:

Average the coordinates to get: $(\frac{3}{2}, -1)$

3. (5 points) Find the equation of the line through the point $(2, 7)$ with slope 3.

Solution:

The slope is 3 and $(2, 7)$ is a point on the line, so the point-slope equation gives:

$$y - 7 = 3(x - 2)$$

In point-intercept form this is:

$$y = 3x + 1$$

4. (5 points) Find the equation of the line through the point $(0, 0)$ parallel to the line $y + 2x = 3$.

Solution:

$$y = -2x$$

5. (20 points) Express the following sets as intervals or unions of intervals:

(a) $\{x \in \mathbb{R} \mid 2 < x < 7\}$

(b) $\{y \in \mathbb{R} \mid -1 \leq y \leq 9\} \setminus \{z \in \mathbb{R} \mid 0 < z < 2\}$

(c) $\mathbb{R} \setminus \{0\}$

(d) $[0, \infty) \cap (-\infty, 2)$

Solution:

(a) $(2, 7)$

(b) $[-1, 0] \cup [2, 9]$

(c) $(-\infty, 0) \cup (0, \infty)$

(d) $[0, 2)$

6. (20 points) For each of the following functions answer these questions:

- What is the domain?
- What is the range?
- Is the function 1 to 1?

(a) $f(x) = 2$

(b) $g(x) = \sqrt{x - 4}$

(c) $h(x) = \sqrt{x^2}$

(d) $k(x) = x^2 \quad , \quad x > 2$

Solution:

(a) Domain is all reals. The range is $\{2\}$, a set consisting of just the number 2. The function is not 1 to 1 since every value of x maps to the number 2.

(b) We must have $x - 4 \geq 0$ so the we are taking the square root of a non-negative number, so the domain is $x \geq 4$. The range is then $[0, \infty)$.

$$\begin{aligned} b = g(a) &= \sqrt{a-4} \\ b^2 &= a - 4 \\ b^2 + 4 &= a \end{aligned}$$

This shows that we can get a number b in the range exactly once, $b = g(a)$ for $a = b^2 + 4$, so the function is 1 to 1.

(c) Domain is all reals. The square root is not a problem since we square first, so we will always be taking the sqaure root of a non-negative number. Range is $[0, \infty)$, since the square root of a non-negative number is a nu=on-negaitve number. The function is not 1 to 1 since for any x we have $h(x) = h(-x) = |x|$.

(d) The domain is $(2, \infty)$, as stated in the definition of the function.

$$\begin{aligned} b = k(a) &= a^2 \\ \pm\sqrt{b} &= a \end{aligned}$$

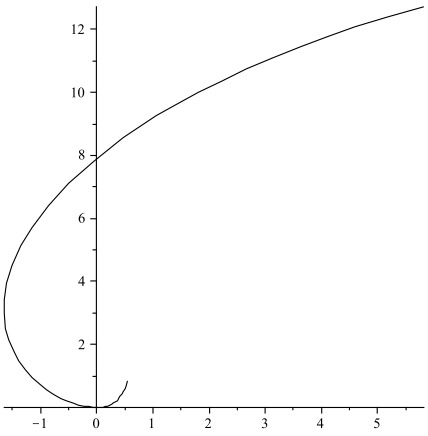
However, we know the domain is $x > 2$ so we should only consider those answers for which $a > 2$. That means first we can throw out the $-\sqrt{b}$. Additionally, we must have $b > 4$ if we are to get $a = \sqrt{b} > 2$. So the range is $(4, \infty)$. The function is 1 to 1 because for $b > 4$ there is only one choice of $a > 2$, $a = \sqrt{b}$, such that $k(a) = b$.

7. (20 points) Do the following formulas or graphs define y as a function of x?

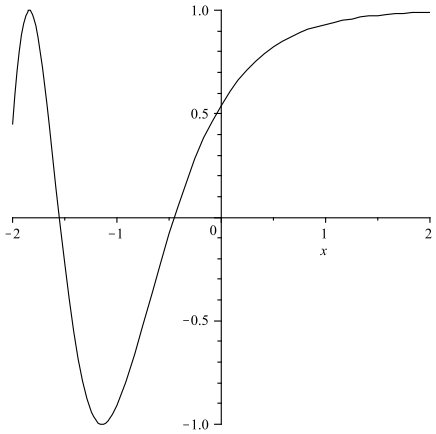
(a) $x^2 + y^2 = 1$

(b) $x + y^2 = 9$

(c)



(d)



Solution:

(a) No. $y = \pm\sqrt{1 - x^2}$, so there are two choices for the y value for a given x.

(b) No. $y = \pm\sqrt{9 - x}$, so there are two choices for the y value for a given x.

(c) No. Graph does not pass the vertical line test. For example, the y-axis passes through two points of the graph.

(d) Yes. Any vertical line passes therough the graph in only one point.

8. (10 points) Is there a line that passes through the three points $(2, 1)$, $(3, -2)$, $(0, 4)$?

Solution:

No. If these points were all on the same line then we could compute the slope of the line by computing the slope between any pair of the points, but the slope using $(2, 1)$ and $(3, -2)$

is -3, while the slope using $(3, -2)$ and $(0, 4)$ is -2. The slopes are different, so the points can not all lie on a line.

9. (10 points) A salesman receives \$1500 per month in base pay, plus 10% of his sales. Write a formula that expresses his monthly pay as a function of his sales. How much will he make if his sales for the month total \$24000.

Solution:

$$Pay(s) = 1500 + .1 * s$$

$Pay(24000) = 3900$

No new questions beyond this point.