MATH 1060 # Exam 1, version A Fall 2008

Solutions

1. (5 points)

(a) Is the following true or false? $(a+b)^2 = a^2 + b^2$

(b) What is the value of the function $f(x) = 2x^2 + 3$ at 3?

Solution:

(a) False. If $a = b = 1$ then $(a+b)^2 = 4$ but $a^2 + b^2 = 2$.

(b) $f(3) = 21$

2. (5 points) Find the midpoint of the line segment joining $(0, 4)$ to $(3, -2)$.

Solution:

Average the coordinates to get: $(\frac{3}{2}, 1)$

3. (5 points) Find the equation of the line through the point $(2, 2)$ with slope 4.

Solution:

The slope is 4 and $(2, 2)$ is a point on the line, so the point-slope equation gives:

$$y - 2 = 4(x - 2)$$

In point-intercept form this is:

$$y = 4x - 6$$

4. (5 points) Find the equation of the line through the point $(0, 0)$ parallel to the line $y + x = 3$.

Solution:

If it is parallel to $y + x = 3$ then it has slope -1, so:

$$y = -x$$

5. (20 points) Express the following sets as intervals or unions of intervals:

(a) $\{x \in \mathbb{R} \mid 2 < x < 7\}$

(b) $\{y \in \mathbb{R} \mid -1 \leq y \leq 9\} \setminus \{z \in \mathbb{R} \mid 0 < z < 2\}$

(c) $\mathbb{R} \setminus \{0\}$

(d) $[0, \infty) \cap (-\infty, 2)$

Solution:

(a) $(2, 7)$

(b) $[-1, 0] \cup [2, 9]$

(c) $(-\infty, 0) \cup (0, \infty)$

(d) $[0, 2)$

6. (20 points) For each of the following functions answer these questions:

- What is the domain?
- What is the range?
- Is the function 1 to 1?

(a) $f(x) = 2x + 3$

(b) $g(x) = \frac{x}{x-1}$

(c) $h(x) = \sqrt{x^2}$

(d) $k(x) = x^2 \quad , \quad x > 2$

Solution:

(a) Domain is $\mathbb{R}$. Suppose $b = f(a) = 2a + 3$, then $a = \frac{b-3}{2}$. This shows that the range is all reals, since for any b we can get b by evaluating f at $a = \frac{b-3}{2}$. This also shows that f is 1 to 1 because for each b there was a single value of a, namely $a = \frac{b-3}{2}$, such that $f(a) = b$.

(b) Domain is $x \neq 1$, since the denomiator would be 0 if $x = 1$. If $b = g(a) = \frac{a}{a-1}$ then:

$$\begin{aligned} b &= \frac{a}{a-1} \\ b(a-1) &= a \\ ba - b &= a \\ ba - a &= b \\ a(b-1) &= b \\ a &= \frac{b}{b-1} \end{aligned}$$

We don't get an a for $b = 1$, because that would make the denominator 0, so the range is $y \neq 1$. The function is 1 to 1 since for any b we got exactly 1 a, $a = \frac{b}{b-1}$, such that $f(a) = b$.

(c) Domain is all reals. There is no issue with the square root because we square first, so the square root always sees a non-negative number.

$$\begin{aligned} b &= \sqrt{a^2} \\ b^2 &= a^2 \\ a &= \pm b \end{aligned}$$

The range is $[0, \infty)$ since the square root of a non-negative number is a non-negative number. The function is not 1 to 1 because we have two choices, $a = \pm b$, such that $h(a) = b$.

(d) The domain is $(2, \infty)$, as stated in the definition of the function.

$$\begin{aligned} b = k(a) &= a^2 \\ \pm\sqrt{b} &= a \end{aligned}$$

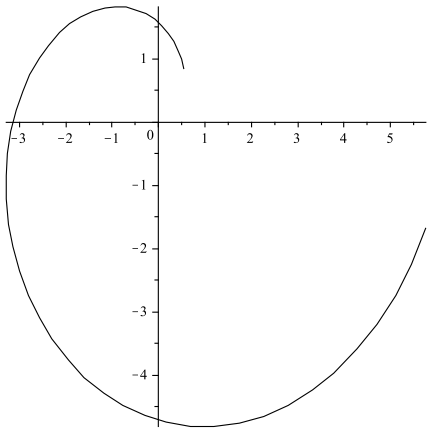
However, we know the domain is $x > 2$ so we should only consider those answers for which $a > 2$. That means first we can throw out the $-\sqrt{b}$. Additionally, we must have $b > 4$ if we are to get $a = \sqrt{b} > 2$. So the range is $(4, \infty)$. The function is 1 to 1 because for $b > 4$ there is only one choice of $a > 2$, $a = \sqrt{b}$, such that $k(a) = b$.

7. (20 points) Do the following formulas or graphs define y as a function of x?

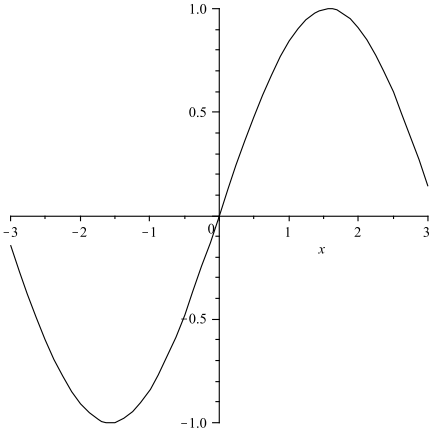
(a) $x^2 + y^2 = 1$

(b) $x^2 + y = 9$

(c)



(d)



Solution:

(a) No. $y = \pm\sqrt{1 - x^2}$, so there are two choices for the y value for a given x.

(b) Yes. $y = 9 - x^2$

(c) No. Graph does not pass the vertical line test. For example, the y-axis passes through two points of the graph.

(d) Yes. Any vertical line passes therough the graph in only one point.

8. (10 points) Is there a line that passes through the three points $(1, 2)$, $(-2, 3)$, $(4, 0)$?

Solution:

No. If these points were all on the same line then we could compute the slope of the line by computing the slope between any pair of the points, but the slope using $(1, 2)$ and $(-2, 3)$ is $-\frac{1}{3}$, while the slope using $(-2, 3)$ and $(4, 0)$ is $-\frac{1}{2}$. The slopes are different, so the points can not all lie on a line.

9. (10 points) A salesman receives \$2000 per month in base pay, plus 8% of his sales. Write a formula that expresses his monthly pay as a function of his sales. How much will he make if his sales for the month total \$24000.

Solution:

$$Pay(s) = 2000 + .08 * s$$

$Pay(24000) = 3920$

No new questions beyond this point.