1. Use induction to prove that

\[ 2^n < 3^n \]

for all \( n \in \mathbb{N} \). You can use all of the usual properties of addition, multiplication and order for the natural numbers (but not properties of exponents).
2. For the following you should assume that $x, y$ and $z$ are elements of a field $F$ as defined in the book and notes.

(a) Prove that if $xz = yz$ and $z \neq 0$ then $x = y$.

(b) Prove that $xy = 0$ then either $x = 0$ or $y = 0$.

In your proofs you can only use the properties of a field given in the notes along with the following two results we proved in class:

(i) If $x + z = y + z$ then $z = y$.
(ii) $x \cdot 0 = 0$. 


3. If $L$ is a Dedekind cut show that the set

$$K = \{ x \in \mathbb{Q} | \exists y \in L \text{ with } x = y + 1 \}$$

is a Dedekind cut.
**Peano Axioms.** The set of natural numbers \( \mathbb{N} \) satisfy the following properties:

N1. There is an element \( 1 \in \mathbb{N} \).

N2. There is a successor function \( s : \mathbb{N} \to \mathbb{N} \).

N3. \( 1 \not\in s(\mathbb{N}) \).

N4. The successor function \( s \) is injective. That is if \( s(n) = s(m) \) then \( n = m \).

N5. If \( A \subset \mathbb{N} \) contains 1 and \( s(A) \subset A \) then \( A = \mathbb{N} \).

**Rings and fields.** A field \( F \) is a set with operations of additions and multiplication that satisfy:

A1. \( x + y = y + x \) for all \( x, y \in F \).

A2. \((x + y) + z = x + (y + z) \) for all \( x, y, z \in F \).

A3. There exists a \( 0 \in F \) such that \( x + 0 = x \) for all \( x \in F \).

A4. For all \( x \in F \) there exists and \( -x \in F \) such that \( x + (-x) = 0 \).

M1. \( xy = yx \) for all \( x, y \in F \).

M2. \((xy)z = x(yz) \) for all \( x, y, z \in F \).

M3. There exists a \( 1 \in F \) with \( 1 \neq 0 \) such that \( x \cdot 1 = x \) for all \( x \in F \).

M4. For all \( x \in F \) with \( x \neq 0 \), there exists an \( x^{-1} \in F \) with \( xx^{-1} = 1 \).

D. \( x(y + z) = xy + xz \) for all \( x, y, z \in F \).

**Dedekind cuts.** A set \( L \subset \mathbb{Q} \) is a Dedekind cut if

(a) \( L \neq \emptyset, \mathbb{Q} \).

(b) There does not exists a \( r \in L \) such that \( r \geq x \) for all \( x \in L \). (\( L \) has no largest element.)

(c) If \( x \in L \) then for all \( y \in \mathbb{Q} \) with \( y < x \), \( y \in L \).
Scratch Work