Midterm 1 notes for Math 3210

Peano Axioms. The set of natural numbers \( \mathbb{N} \) satisfy the following properties:

N1. There is an element 1 \( \in \mathbb{N} \).
N2. There is a successor function \( s : \mathbb{N} \to \mathbb{N} \).
N3. 1 \( \not\in s(\mathbb{N}) \).
N4. The successor function \( s \) is injective. That is if \( s(n) = s(m) \) then \( n = m \).
N5. If \( A \subset \mathbb{N} \) contains 1 and \( s(A) \subset A \) then \( A = \mathbb{N} \).

Rings and fields. A field \( F \) is a set with operations of additions and multiplication that satisfy:

A1. \( x + y = y + x \) for all \( x, y \in F \).
A2. \( (x + y) + z = x + (y + z) \) for all \( x, y, z \in F \).
A3. There exists a 0 \( \in F \) such that \( x + 0 = x \) for all \( x \in F \).
A4. For all \( x \in F \) there exists and \( -x \in F \) such that \( x + (-x) = 0 \).
M1. \( xy = yx \) for all \( x, y \in F \).
M2. \( (xy)z = x(yz) \) for all \( x, y, z \in F \).
M3. There exists a 1 \( \in F \) such that \( x \cdot 1 = x \) for all \( x \in F \).
M4. For all \( x \in F \) with \( x \neq 0 \), there exists an \( x^{-1} \in F \) with \( xx^{-1} = 1 \).
D. \( x(y + z) = xy + xz \) for all \( x, y, z \in F \).

Dedekind cuts. A set \( L \subset \mathbb{Q} \) is a Dedekind cut if
(a) \( L \neq \emptyset, \mathbb{Q} \).
(b) There does not exists a \( r \in L \) such that \( r \geq x \) for all \( x \in L \). (\( L \) has no largest element.)
(c) If \( x \in L \) then for all \( y \in \mathbb{Q} \) with \( y < x \), \( y \in L \).