2.1.2 If $|x - 1| < 1/2$ and $|x - 2| < 1/2$ then

$$1 = 1/2 + 1/2 > |x - 1| + |x - 2| = |x - 1| + |x - 2| \geq |x - 1 + 2 - 1| = 1.$$  

Since $1 > 1$ is a contradiction we can’t have that $|x - 1| < 1/2$ and $|x - 2| < 1/2$.

2.1.5 We claim that

$$\lim_{n \to \infty} \frac{2n - 1}{3n + 1} = \frac{2}{3}.$$  

Fix $\epsilon > 0$. Choose $N \in \mathbb{N}$ such $N > \frac{5}{9\epsilon}$. Then if $n \geq N$ we have

$$\left| \frac{2n - 1}{3n + 1} - \frac{2}{3} \right| = \frac{5}{3(3n + 1)} < \frac{5}{9n} < \epsilon$$

since if $n > N$ then $\frac{5}{9n} < \epsilon$. Therefore

$$\frac{2n - 1}{3n + 1} \to \frac{2}{3}.$$  

2.1.6 We claim that $\lim_{n \to \infty} \frac{(-1)^n}{n} = 0$. Fix $\epsilon > 0$. Choose $N \in \mathbb{N}$ such that $N > \frac{1}{\epsilon}$. Then if $n \geq N$ we have

$$\left| \frac{(-1)^n}{n} - 0 \right| = \frac{1}{n} < \epsilon$$

since if $n > N > \frac{1}{\epsilon}$ then $\frac{1}{n} < \epsilon$. Therefore $\frac{(-1)^n}{n} \to 0$.

2.1.8 We claim that $\lim_{n \to \infty} \sqrt{n+1} - \sqrt{n} = 0$. Fix $\epsilon > 0$ and choose $N \in \mathbb{N}$ such that $N > \frac{1}{4\epsilon^2}$. Note that if $n > N$ then $n + 1 > \frac{1}{4\epsilon^2}$ which implies that $\epsilon > \frac{1}{2\sqrt{n+1}}$. Therefore

$$\left| \sqrt{n+1} - \sqrt{n} - 0 \right| = \left| \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}} \right|$$

$$= \left| \frac{n+1 - n}{\sqrt{n+1} + \sqrt{n}} \right| \leq \frac{1}{2\sqrt{n+1}} < \epsilon$$
\[ \sqrt{n + 1} - \sqrt{n} \to 0. \]

2.1.10 We first show that \(2^n \geq n\) by induction. When \(n = 1\) we have that \(2^1 = 2 \geq 1\).
Now we assume that \(2^n \geq n\) and show that \(2^{n+1} \geq n + 1\). For this we see that
\[
2^{n+1} = 2^n + 2^n \geq n + n \geq n + 1.
\]
We have proved the induction step and it follows that \(2^n \geq n\) for all \(n\).
Now fix \(\epsilon > 0\) and choose \(N \in \mathbb{N}\) such that \(N > \frac{1}{\epsilon}\). Then if \(n > N\) we have
\[
|2^{-n} - 0| = 1/2^n \leq 1/n \leq 1/N < \epsilon
\]
and \(2^{-n} \to 0\).