- **Product rule:** If $F(x) = f(x)g(x)$ then

$$F'(x) = f'(x)g(x) + f(x)g'(x).$$

- **Quotient rule:** If $F(x) = \frac{f(x)}{g(x)}$ then

$$F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

- **Chain rule:** If $F(x) = f(g(x))$ then

$$F'(x) = f'(g(x))g'(x).$$

- **Power rule:** If $F(x) = f(x)^n$ then

$$F'(x) = nf(x)^{n-1}f'(x).$$

- **Quadratic formula:** If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- **Integrals:**

$$\int f'(x)f(x)^n dx = \frac{f(x)^{n+1}}{n+1} + C \text{ if } n \neq -1;$$

$$\int f'(x)f(x)^{-1} dx = \ln|f(x)| + C;$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + C.$$

- **Elasticity:** If q is the quantity and p is the price then the elasticity is

$$-\frac{p}{q}\frac{dq}{dp}.$$

- **Present value:** If $f(t)$ is a revenue stream and r the interest then the future value of the stream after T years is

$$\int_0^T f(t)e^{-rt} dt.$$

(1) Find the derivative $\dfrac{dy}{dx}$ if

   (a) $y = xe^{x^2}$
   (b) $y = \ln(x^2+3)$
   (c) $y = \dfrac{1}{\sqrt{x^2+1}}$
   (d) $y = \dfrac{e^{\sqrt{x}}}{1+x}$
   (e) $y = \ln(x(x^2+1))$
   (f) $y = e^{3x^2+1}$

(2) Find the second derivative of $f(x) = \frac{x}{3x+1}$.

(3) Find the equation of the tangent line to $y^3 = x^2 - 3$ at $(x, y) = (-2, 1)$.

(4) Let $f(x) = \frac{4x^2+1}{3x^2+1}$. The first derivative of f is $f'(x) = \frac{2x}{(3x^2+1)^2}$ and the second derivative is $f''(x) = \frac{2(1-9x^2)}{(3x^2+1)^3}$.
   (a) Find the critical values of f.
   (b) Find the intervals where f is increasing and the intervals where f is decreasing.
   (c) Find the intervals where f is concave up and the intervals where f is concave down.
   (d) Find x-values for any inflection points for f.
   (e) Find the x-values of any relative maxima and minima. Make sure you clear which values correspond to maxima and which to minima.
   (f) Find the horizontal and vertical asymptotes.

(5) If $f(x, y) = \dfrac{x}{y-x}$, determine the following:

   (a) $\dfrac{\partial f}{\partial x}$
   (b) $\dfrac{\partial^2 f}{\partial x \partial y}$

(6) Compute the following integrals:

   (a) $\displaystyle\int \frac{3x}{x^2+1}\,dx$
   (b) $\displaystyle\int \frac{x}{(x^2+2)^2}\,dx$
   (c) $\displaystyle\int 8x\sqrt{x^2-1}\,dx$
   (d) $\displaystyle\int_0^1 \frac{x}{e^{x^2}}\,dx$
   (e) $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx$

(7) Suppose that a donor wishes to provide a cash gift to the University that will generate a continuous income stream with an annual rate of flow at time t given by $f(t) = \$80,000$ per year. If the annual interest rate of 4% compounded continuously, find the capital value of this income stream.

(8) A company has an income stream of $f(t) = 7000e^{-.03t}$ in dollars per year. Find the present value of the income stream over the next 10 years if it is invested at a rate of 2% a year compounded continuously. Also find the capital value of the income stream given the same rate of interest.

(9) Find the area of the region between the curves $y = x^2 - 1$ and $y = -x - 1$.

(10) A company's profit from selling pens and pencils is

$$P(x, y) = 3x + 2y - .01x^2 - .02y^2 - .003xy$$

where x is the number of pens and y is the number of pencils. If 20 pens and 10 pencils have been sold what is the approximate amount of profit from selling the 11th pencil?

(11) The marginal cost function for an item is $\overline{MC}(x) = 100 + 3x$ and the marginal revenue function is $\overline{MR}(x) = 300 - x$. The initial cost is \$50. Find the maximum profit or minimal loss.

(12) The profit function for an item is $P(x) = 9x - x^3 - 3x^2 - 2$ and you are only allowed to make up to 10 items. Find the maximum profit or minimal loss.

(13) The profit function for an item is $P(x) = x + \dfrac{100}{x-20} + 2$ and you are only allowed to make up to 15 items. Find the maximum profit or minimal loss.

(14) The demand function for a certain item is given by

$$(p+1)(q+6)^2 = 1000\,.$$

Find the elasticity of demand when $p = 9$. If the price is increased will revenue increase or decrease?

(15) In 1988 the Lorenz curve for income distribution in the U.S. was $y = x^{2.3521}$. What was the Gini coefficient? If the Gini Coefficient in another country that some year was .4 was the income distribution more equal in this country or in the U.S.

(16) Find the consumer's surplus at the equilibrium price given that the demand function is $p = -x^2 - 6x + 75$ and the supply function is $p = 2x + 10$.

(17) Find the consumer's surplus at the equilibrium price given that the demand function is $p = -x + 170$ and the supply function is $p = x^2 + 4x + 20$.