1. Prove the Jacobi identity:

\[
[[v_1, v_2], v_3] + [[v_2, v_3], v_1] + [[v_3, v_1], v_2] = 0
\]

2. Let \(v_1, \ldots, v_k\) be vector fields defined in a neighborhood of 0 in \(\mathbb{R}^n\). Let \(\phi^j_i\) be the respective flows. Define

\[
\chi(x_1, \ldots, x_n) = \phi^1_{x_1} \circ \phi^2_{x_2} \circ \cdots \circ \phi^k_{x_k}(0, \ldots, 0, x_{k+1}, \ldots, x_n)
\]

If \(v_1(0), \ldots, v_k(0)\) are linearly independent show that \(d\chi_0\) is an isomorphism and that \(\chi^*(\frac{\partial}{\partial x_1}) = v_1\).