Bifurcation theory: Problems I

[1.1] Prove that the system

$$\dot{x}_1 = -x_1, \quad \dot{x}_2 = -x_2$$

is topologically equivalent near the origin to the system

$$\dot{x}_1 = -x_1, \quad \dot{x}_2 = -2x_2$$

(Hint: mimic the proof of the equivalence beteen a node and a focus without introducing polar coordinates). Are the systems diffeomorphic?

[1.2] Show that the scalar system

$$\dot{y} = \beta y - y^2$$

which exhibits the transcritical bifurcation is topologically equivalent (in fact, diffeomorphic) to a system **induced** by the system

$$\dot{x} = \alpha - x^2$$

which undergoes the saddle–node bifurcation. [Note: induced means that the mapping $\alpha = p(\beta)$ may not be invertible].

[1.3] Reduce the following systems to an equation on the extended center manifold (up to cubic order), identify the bifurcation and sketch the bifurcation diagram:

(a)

$$\dot{x} = -\frac{1}{2}\mu(x+y) + y + \frac{5}{16}x^2 + \frac{xy}{8} - \frac{3y^2}{16}$$
$$\dot{y} = -\frac{1}{2}\mu(x+y) + x + \frac{3}{16}x^2 - \frac{xy}{8} - \frac{5y^2}{16}$$

(b)

$$\dot{x} = y - x - x^2$$
$$\dot{y} = \mu x - y - y^2$$