5.2

4) Let $c_1 = -1/17$, $c_2 = 1/2$ and $c_3 = 1/3$ so that

$$c_1 17 + c_2 2\sin^2(x) + c_3 3\cos^2(x) = -1 + 1 = 0$$

10)

$$W = \begin{vmatrix} e^x & \frac{1}{x^2} & \frac{\ln(x)}{x^2} \\ e^x & \frac{-1}{x^3} & \frac{1}{x^3} - \frac{2\ln(x)}{x^3} \\ e^x & \frac{1}{x^4} & \frac{-2}{x^4} + \frac{6\ln(x)}{x^4} \end{vmatrix} = \frac{e^x(x+4)(x+1)}{x^7}$$

which is nonzero for $x > 0$.

18) The general solution is $y = c_1 e^x + c_2 e^x \cos(x) + c_3 \sin(x)$. Using the initial conditions we get the three equations for c_1, c_2, c_3

$$\begin{aligned} y(0) &= c_1 + c_2 = 1 \\ y'(0) &= c_1 + c_2 + c_3 = 0 \\ y''(0) &= c_1 + 3c_3 = 0 \end{aligned}$$

Solving yields $c_1 = 2, c_2 = -1, c_3 = -1$ giving the particular solution $y = e^x(2 - \cos(x) - \sin(x))$.

22) We add the complementary solution, $y_c = c_1 e^{2x} + c_2 e^{-2x}$, and particular solution, $y_p = -3$ to get the general solution to the inhomogeneous problem, $y = y_c + y_p = c_1 e^{2x} + c_2 e^{-2x} - 3$. Now we use the initial conditions to get the final solution. This yields the two equations for c_1, c_2, $c_1 + c_2 = 3$ and $2c_1 - 2c_2 = 0$, which gives us $c_1 = 4, c_2 = -1$ and

$$y = 4e^{2x} - e^{-2x} - 3$$

5.3

6) We get the characteristic equation $r^2 + 5r + 5 = 0$, which means we have the two real roots $r = -\frac{5}{2} \pm \frac{\sqrt{5}}{2}$. The general solution is then $y = c_1 e^{(-\frac{5}{2}+\frac{\sqrt{5}}{2})x} + c_2 e^{(-\frac{5}{2}-\frac{\sqrt{5}}{2})x}$.

8) We get the characteristic equation $r^2 - 6r + 13 = 0$, which means we have the two complex roots $r = 3 \pm 2i$. The general solution is then $y = c_1 e^{(3+2i)x} + c_2 e^{(3-2i)x} = e^{3x}(C_1\cos(2x) + C_2\sin(2x))$.

14) We get the characteristic equation $r^4 + 3r^2 - 4 = (r^2-1)(r^2+4) = 0$, which means we have the four roots $r = \pm 1, \pm 2i$. The general solution is then $y = c_1 e^{2ix} + c_2 e^{-2ix} + c_3 e^x + c_4 e^{-x} = C_1\cos(2x) + C_2\sin(2x) + c_3 e^x + c_4 e^{-x}$.

24) We get the characteristic equation $2r^3 - 3r^2 - 2r = r(2r^2 - 3r - 2) = r(2r+1)(r-2) = 0$, which means we have the three real roots $r = 0, -1/2, 2$. The general solution is then $y = c_1 + c_2 e^{-1/2x} + c_3 e^{2x}$. Using the initial conditions we get the three equations for c_1, c_2, c_3 as $y(0) = c_1 + c_2 + c_3 = 1$, $y'(0) = c_1 - 1/2c_2 + 2c_3 = -1$, and $y''(0) = c_1 + 1/4c_2 + 4c_3 = 3$ so that $c_1 = -7/2, c_2 = 1/2, c_3 = 4$ gives us the particular solution

$$y = 1/2(-7 + e^{-1/2x} + 8e^{2x}).$$

36) Given the solution $e^{-x}\sin(x)$ we know a root of the characteristic equation $9r^3 + 11r^2 + 4r - 14 = 0$ is $r = -1 - i$, but since (with real coefficients of the ODE) we have the complex conjugate, $-1 + i$ as a root as well, $r^2 + 2r + 2$ is a factor of the characteristic equation. Either using polynomial long division or letting $(r - r_0)$ be the other factor, expanding, and comparing coefficients we get that the third root (which must be real) is $r = 7/9$. So the general solution is

$$y = c_1 e^{7/9x} + e^{-x}(c_2\cos(x) + c_3\sin(x))$$

42) We know from the form of the solution the roots $\pm 2i$ have a multiplicity of 3 (giving us 6 roots so that we expect a 6th order ODE), so the characteristic equation when factored looks like $(r^2+4)^3 = 0$. Expanding the characteristic equation we get

$$r^6 + 12r^4 + 48r^2 + 64 = 0.$$

Since each power of r relates to a derivative, we get the the 6th order ODE

$$y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0$$

5.4

8) Let n be the number of cycles in of the pendulum required for the clock to register 24hrs = 1440 min. Then its period with length $L = 30$in is $p_1 = 1430/n$ min (i.e., 10 min slower than 24hrs.). The circular frequency of the pendulum is given by $\sqrt{\frac{g}{L}}$ so that

$$1430/n = \sqrt{\frac{30}{g}}.$$

We wish to solve for the length L_2 such that

$$1440/n = \sqrt{\frac{L_2}{g}}.$$

Using these two equations we get that

$$L_2 = 30(1440/1430) = 30.42in.$$

16) Since $c^2 = 900 > 756 = 4km$, we are in an overdamped situation. The roots are $r = \frac{-c \pm \sqrt{c^2-4km}}{2m} = \frac{-30 \pm \sqrt{900-756}}{6} = -3, -7$, so that the general solution is $x = c_1 e^{-3t} + c_2 e^{-7t}$. Using the initial conditions we get the particular solution $x = 4e^{-3t} - 2e^{-7t}$.

18) Since $c^2 = 144 < 400 = 4km$, we are in an underdamped situation. The roots are $r = \frac{-c \pm \sqrt{c^2-4km}}{2m} = \frac{-12 \pm \sqrt{144-400}}{4} = -3 \pm 4i$, so that the general solution is $x = e^{-3t}(c_1 \cos(4t) + c_2 \sin(4t)$. Using the initial conditions we get the particular solution $x = -2e^{-3t} \sin(4t)$.