4.5

2) We can check both closure under addition and scalar multiplication at once by checking if a linear combination of symmetric matrices are symmetric. A matrix is symmetric if $A = A^T$. Check is $cA + dB$ is also symmetric is A and B are symmetric.

$$(cA + dB)^T = cA^T + dB^T = cA + dB$$

So the linear combination is also symmetric and so the set of all symmetric three by three matrices is a subspace.

8) Again checking a linear combination of odd functions we get

$$(af + bg)(-x) = af(-x) + bg(-x) = a(-f(x)) + b(-g(x)) = -(af + bg)(x)$$

So the odd functions form a subspace.

10)

$$c(a_2x^2 + a_3x^3) + d(b_2x^2 + b_3x^3) = (ca_2 + db_2)x^2 + (ca_3 + db_3)x^3$$

so that the set polynomials of at most degree 3 without the constant and linear terms is a subspace.

14) Either $xe^x = ke^x$ or $e^x = kxe^x$ for some constant k for xe^x and e^x to be linearly dependent, but $k \neq x$ and $k \neq 1/x$ so that xe^x and e^x are linearly independent.

16) Looking for constants c_1, c_2, c_3 such that $c_1(1 + x) + c_2(x + x^2) + c_3(1 - x^2) = 0$ we find by equating coefficients that $c_1 = -1, c_2 = 1, c_2 = 1$ satisfy the equation so that $1 + x, x + x^2, 1 - x^2$ are linearly dependent.

26) Let $y'(x) = v(x)$. Then $v(x)$ solves the first order equation $v' + 10v = 0$. Using an integrating factor (or separation of variables) we get the general solution $v(x) = Ce^{-10x}$. Then $y' = Ce^{-10x}$ and $y = C_1e^{-10x} + C_2$. The solution space is 2-dimensional with basis $\{e^{-10x}, 1\}$.

5.1

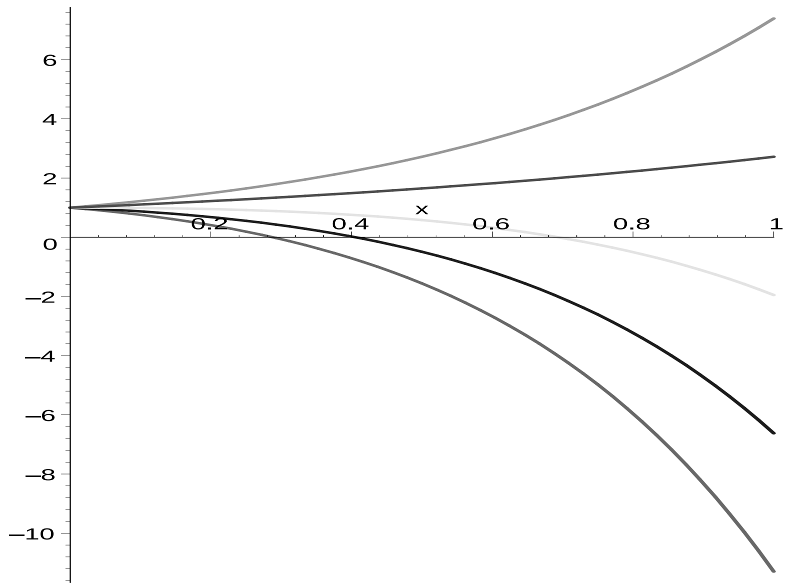
8) Using the initial conditions $y(0) = 4$ and $y'(0) = -2$ to get particular solution $y = 14/3 - 2/3e^{3x}$.

26) Either $2\cos(x) + 3\sin(x) = k(3\cos(x) - 2\sin(x))$ or $3\cos(x) - 2\sin(x) = k(2\cos(x) + 3\sin(x))$ for all values of x. Check at $x = 0$ and $x = \pi/2$. This shows $2\cos(x) + 3\sin(x), 3\cos(x) - 2\sin(x)$ are independent.

34) Roots of the characteristic equation are $r = 3, 5$ and the general solution is $y = C_1e^{3x} + C_2e^{5x}$.

40) The root of the characteristic equation is $r = 2/3$ and the general solution is $y = C_1e^{2/3x} + C_2xe^{2/3x}$.

CP2) The general solution is $y = C_1 e^x + C_2 e^{2x}$. If we let $y(0) = 1$ and $y'(0) = b$, then $y = (2-b)e^x + (b-1)e^{2x}$ yielding different curves for choices of $b = -2, -1, 0, 1, 2$.



If we let $y(0) = b$ and $y'(0) = 1$, then $y = (2b-1)e^x + (1-b)e^{2x}$ yielding different curves for choices of $b = -2, -1, 0, 1, 2$.