4.3 4) Four vectors in R^3 must be linearly dependent since we only need three to describe all of R^3.

12) Can we find c_1 and c_2 such that $c_1 v_1 + c_2 v_2 = w$?

$$\left[\begin{array}{cc|c} 7 & -2 & 4 \\ 3 & -2 & -4 \\ -1 & 1 & 3 \\ 9 & -3 & 3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

so that $c_1 = 2$ and $c_2 = 5$ and $2v_1 + 5v_2 = w$.

14) Can we find c_1, c_2 and c_3 such that $c_1 v_1 + c_2 v_2 + c_3 v_3 = w$?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -2 & 1 & 2 \\ 3 & 0 & 1 & -3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

so that the system is inconsistent and there don't not exist such a c_1, c_2 and c_3.

22) Can we find c_1, c_2 and c_3 such that $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$?

$$\left[\begin{array}{ccc|c} 3 & 3 & 5 & 0 \\ 9 & 0 & 7 & 0 \\ 0 & 9 & 5 & 0 \\ 5 & -7 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 7/9 & 0 \\ 0 & 1 & 5/9 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

We have 1 free parameter and therefore a 1-dimensional solution space. Choosing $c_3 = -9$, then $c_2 = 5$ and $c_1 = 7$.

33) The determinant of the $k \times k$ identity matrix is $1 \neq 0$ so that from Theorem 3 of this section we are guaranteed that the vectors are linearly independent.

4.4 6) $\det([v_1 v_2 v_3]) = -1 \neq 0$ so the vectors are linearly independent.

10) We can write $y = z$ in reduced echelon form as

$$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

so there are two free variables $x = s$ and $z = t$ and solution vector is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

So our two basis vectors are $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

12) Since $a = b + c + d$ the solution vector is

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} b+c+d \\ b \\ c \\ d \end{pmatrix} = b \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + d \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

using b, c, d as the free parameters. So our three basis vectors are $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.

20)

$$\left[\begin{array}{cccc|c} 1 & -3 & -10 & 5 & 0 \\ 1 & 4 & 11 & -2 & 0 \\ 1 & 3 & 8 & -1 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

leaving us the free variables $x_4 = t, x_3 = s$, so that $x_2 = -3s + t$ and $x_1 = s - 2t$. The solution is then

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} s - 2t \\ -3s + t \\ s \\ t \end{pmatrix} = s \begin{pmatrix} 1 \\ -3 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

So our basis vectors are $\begin{pmatrix} 1 \\ -3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

24)

$$\begin{bmatrix} 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \\ 2 & 7 & -10 & -19 & 13 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

leaving us the free variables $x_5 = t, x_4 = s, x_3 = r$, so that $x_2 = 2r + 3s - t$ and $x_1 = -2r - s - 3t$. The solution is then

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2r - s - 3t \\ 2r + 3s - t \\ r \\ s \\ t \end{pmatrix} = r \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ 3 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

So our basis vectors are $\begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.