3.5 6) $D = 4 \cdot 6 - 3 \cdot 7 = 3 \neq 0$ so that

$$A^{-1} = 1/D \begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix}$$

To solve $Ax = b$, we multiply both sides by A^{-1} to get

$$x = A^{-1}b = 1/D \begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \end{bmatrix} = 1/D \begin{bmatrix} 25 \\ -10 \end{bmatrix}$$

16) We find A^{-1} by augmenting the coefficient matrix A by the identity and then row reducing A to the identity matrix. This may be done using the following elementary row operations

$$\begin{aligned} R_1 + R_2 &\rightarrow R_2 \\ -2R_1 + R_3 &\rightarrow R_3 \\ R_2 + R_3 &\rightarrow R_2 \\ -3R_2 + R_3 &\rightarrow R_3 \\ -1/3R_3 &\rightarrow R_3 \\ -2R_3 + R_2 &\rightarrow R_2 \\ 3R_3 + R_1 &\rightarrow R_1 \\ 3R_2 + R_1 &\rightarrow R_1 \end{aligned}$$

$$A^{-1} = 1/3 \begin{bmatrix} -3 & 0 & 3 \\ -1 & -3 & -1 \\ -1 & 3 & 2 \end{bmatrix}$$

26) To find a solution to $AX = B$ we find A^{-1} and multiply both sides (on the left) by A^{-1}.

$$A^{-1} = \begin{bmatrix} -16 & 3 & 11 \\ 6 & -1 & -4 \\ -13 & 2 & 9 \end{bmatrix}$$

so that

$$X = \begin{bmatrix} -16 & 3 & 11 \\ 6 & -1 & -4 \\ -13 & 2 & 9 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -21 & 9 & 6 \\ 8 & -3 & -2 \\ -17 & 6 & 5 \end{bmatrix}$$

30) If ABC is invertible, there exists a matrix D such that

$$ABCD = I.$$

Since A is invertible

$$BCD = A^{-1}.$$

Since B is invertible

$$CD = B^{-1}A^{-1}.$$

Since C is invertible,

$$D = C^{-1}B^{-1}A^{-1}$$

being careful to multiply on the appropriate side of the expression on both sides of the equation. Thus, D exists and $D = C^{-1}B^{-1}A^{-1}$. (You can also check that $C^{-1}B^{-1}A^{-1}$ is the inverse of A by multiplying $(ABC)(C^{-1}B^{-1}A^{-1})$ and getting I.)

32) If A is invertible, we can multiply both sides of $AB = AC$ to get $A^{-1}AB = A^{-1}AC$ which implies $IB = IC$ and $B = C$.

3.6 6) Expanding around the middle row we get

$$\begin{vmatrix} 3 & 0 & 11 & -5 & 0 \\ -2 & 4 & 13 & 6 & 5 \\ 0 & 0 & 5 & 0 & 0 \\ 7 & 6 & -9 & 17 & 7 \\ 0 & 0 & 8 & 2 & 0 \end{vmatrix} = 5 \begin{vmatrix} 3 & 0 & -5 & 0 \\ -2 & 4 & 6 & 5 \\ 7 & 6 & 17 & 7 \\ 0 & 0 & 2 & 0 \end{vmatrix}.$$

Expanding the submatrix around the last row we get

$$5\begin{vmatrix} 3 & 0 & -5 & 0 \\ -2 & 4 & 6 & 5 \\ 7 & 6 & 17 & 7 \\ 0 & 0 & 2 & 0 \end{vmatrix} = 5\cdot(-2)\begin{vmatrix} 3 & 0 & 0 \\ -2 & 4 & 5 \\ 7 & 6 & 7 \end{vmatrix}$$

with the negative coming because to 2 is in 4,3 element. Expanding the submatrix around the first row we get

$$5\cdot(-2)\begin{vmatrix} 3 & 0 & 0 \\ -2 & 4 & 5 \\ 7 & 6 & 7 \end{vmatrix} = 5\cdot(-2)\cdot 3\begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix} = 60$$

12) Performing the row operation $2R_1 + R_4 \to R_4$ puts the matrix into triangular form so that the determinant is $2\cdot 1\cdot 5\cdot 1 = 10$.

16) We use the row operations

$$\begin{aligned} R_3 + R_2 &\to R_2 \\ 2R_3 + R_1 &\to R_1 \\ 4R_2 + R_1 &\to R_1 \end{aligned}$$

to get a triangular matrix with determinant 84.

18) We use the row operation

$$-3R_1 + R_3 \to R_3$$

and then expand around the first column followed by

$$\begin{aligned} 9R_1 + R_2 &\to R_2 \\ -R_1 + R_3 &\to R_3 \end{aligned}$$

on the submatrix followed the the evaluation of the 2×2 determninat to get that the determinant is 135.

50) Taking the determinant of $A^2 = A$ yields $|A^2| = |A|$ or $|A\cdot A| = |A|$ The product of determinants theorem tells us that this is the same as $|A|\cdot|A| = |A|$ or $|A|(|A|-1|) = 0$ so that $|A| = 0$ or 1.

52) We know from the theorems that $|A| = |A^T|$ and $|A^1| = 1/|A|$. If A is orthogonal then $|A^T| = |A^{-1}|$ so that $|A| = 1/|A|$ or $|A| = \pm 1$.

53) If $A = P^{-1}BP$, then $|A| = |P^{-1}BP| = |P^{-1}||B||P| = 1/|P||B||P| = |B|1/|P||P| = |B|$.