3.1

6) The following manipulations give the solution set.

$$\begin{aligned} \frac{1}{2}E_1 &\rightarrow E_1 \\ -3E_1 + E_2 &\rightarrow E_2 \end{aligned}$$

This show that the system is inconsistent and there is no solution.

14) The following manipulations give the solution set.

$$\begin{aligned} E_1 - E_2 &\rightarrow E_1(putsa1asthefirstelement) \\ -3E_1 + E_2 &\rightarrow E_2 \\ -2E_1 + E_3 &\rightarrow E_3 \\ 9E_2 - 23E_3 &\rightarrow E_3 \end{aligned}$$

To get $z = -4$ and back substitution gives $y = 3$ and $x = 5$.

22) The following manipulations give the solution set.

$$\begin{aligned} E_1 &\leftrightarrow E_2(putsa1asthefirstelement) \\ -4E_1 + E_2 &\rightarrow E_2 \\ -2E_1 + E_3 &\rightarrow E_3 \end{aligned}$$

The second row is a multiple of the third row so that we pick z to be arbitrary $z = t$. Back substituting gives $y = -5t$ and $x = -4t$ for some $t \in R$.

26) Plugging in the two initial conditions yields the system

$$\begin{aligned} A + B &= 44 \\ 11A - 11B &= 22 \end{aligned}$$

The following manipulations give the solution set.

$$\begin{aligned} 1/11E_2 &\rightarrow E_2 \\ -E_1 + E_2 &\rightarrow E_2 \\ -1/2E_2 &\rightarrow E_2 \end{aligned}$$

This shows that $B = 21$ and $A = 23$. The particular solution is $y(x) = 23e^{11x} + 21e^{-11x}$.

3.2

14) We row reduce the following augmented matrix

$$\left[\begin{array}{ccc|c} 3 & -6 & -2 & 1 \\ 2 & -4 & 1 & 17 \\ 1 & -2 & -2 & -9 \end{array}\right]$$

using the following elementary row operations.

$$\begin{aligned} R_1 &\leftrightarrow R_3 \\ -2R_1 + R_2 &\rightarrow R_2 \\ -3R_1 + R_3 &\rightarrow R_3 \end{aligned}$$

We see that $x_3 = 7$ and that R_3 is a multiple of R_2, so we pick the free variable $x_2 = t$. Back substituting we get that $x_1 = 2t + 5$.

20) We row reduce the following augmented matrix

$$\left[\begin{array}{ccccc|c} 2 & 4 & -1 & -2 & 2 & 6 \\ 1 & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{array}\right]$$

using the following elementary row operations.

$$\begin{aligned} R_1 &\leftrightarrow R_2 \\ -2R_1 + R_2 &\rightarrow R_2 \\ -5R_1 + R_3 &\rightarrow R_3 \\ -4R_2 + R_3 &\rightarrow R_3 \\ R_2 &\leftrightarrow R_3 \\ 2R_2 + R_3 &\rightarrow R_3 \end{aligned}$$

which yields the echelon matrix

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & -7 & 3 & 9 \\ 0 & 1 & 3 & -7 & 2 & 7 \\ 0 & 0 & 1 & -2 & 0 & 2 \end{array}\right]$$

The free variables are $x_4 = s$ and $x_5 = t$. Then $x_3 = 2s + 2$, $x_2 = 1 + s - 2t$, and $x_1 = 2 + 3t$.

28) Using elementary row operations we can reduce

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & a \\ 1 & 2 & 1 & b \\ 7 & 4 & 9 & c \end{array}\right]$$

to

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & b \\ 0 & -5 & 1 & a - 2b \\ 0 & 0 & 0 & c - 2a - 3b \end{array}\right].$$

If $c - 2a - 3b = 0$, then we have a row of zeros, and we get that $z = t$ is a free variables and an infinite number of solutions $(-\frac{7}{5}t - 2a + 5b, \frac{1}{5}t + a - 2b, t)$. If $c - 2a - 3b \neq 0$, then we have an inconsistent system with no solution.

3.3

8) We take the matrix

$$\begin{bmatrix} 1 & -4 & -5 \\ 3 & -9 & 3 \\ 1 & -2 & 3 \end{bmatrix}$$

and using the following elementary row operations

$$\begin{aligned} -3R_1 + R_2 &\rightarrow R_2 \\ -R_1 + R_3 &\rightarrow R_3 \\ R_2 - R_3 &\rightarrow R_2 \\ -2R_2 + R_3 &\rightarrow R_3 \\ -1/12R_3 &\rightarrow R_3 \\ -10R_3 + R_2 &\rightarrow R_2 \\ 5R_3 + R_1 &\rightarrow R_1 \\ 4R_2 + R_1 &\rightarrow R_1 \end{aligned}$$

which yields the reduced echelon matrix of the identity matrix.

16) We take the 3×4 matrix

$$\begin{bmatrix} 1 & 3 & 15 & 7 \\ 2 & 4 & 22 & 8 \\ 2 & 7 & 34 & 17 \end{bmatrix}$$

and using the following elementary row operations

$$\begin{aligned} -2R_1 + R_2 &\rightarrow R_2 \\ -2R_1 + R_3 &\rightarrow R_3 \\ -1/2R_2 &\rightarrow R_2 \\ -R_2 + R_3 &\rightarrow R_3 \\ -3R_2 + R_1 &\rightarrow R_1 \end{aligned}$$

which yields the echelon matrix

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

32) We reduce the general 2×2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

using the following elementary row operations. If $D = ad - bc \neq 0$, then a and c cannot both be zero. If $a \neq 0$, then we use the following row operations

$$\begin{aligned} 1/aR_1 &\to R_1 \\ -cR_1 + R_2 &\to R_2 \\ aR_2 &\to R_2 \\ 1/DR_2 &\to R_2 \\ -b/aR_2 + R_1 &\to R_1 \end{aligned}$$

which yields the reduced echelon matrix of the identity matrix. Similar calculation may be done assuming $c \neq 0$.

3.4 8) AB =

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -5 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 15 \\ 35 & 0 \end{bmatrix}$$

BA =

$$\begin{bmatrix} 3 & 0 \\ -1 & 4 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 2 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 9 \\ 7 & -20 & 13 \\ 16 & -25 & 38 \end{bmatrix}$$

12) Since A is (1×4) and B is (2×3), neither AB nor BA is defined.

20) We write the linear system as the matrix equation

$$\begin{bmatrix} 1 & -3 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since the coefficient matrix is in reduced echelon form we can ascribe the free variables to $x_5 = t$ and $x_2 = s$ and write down the solution as a linear combination of vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -7 \\ 0 \\ 2 \\ 10 \\ 0 \end{bmatrix}$$

32) If

$$A = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}; \; B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix},$$

then

$$(A+B)^2 = \begin{bmatrix} 5 & 52 \\ -13 & 96 \end{bmatrix}$$

and

$$A^2 + 2AB + B^2 = \begin{bmatrix} 22 & 41 \\ 14 & 79 \end{bmatrix}$$

b) Assuming $AB = BA$, then using the distributive property

$$\begin{aligned} (A+B)^2 &= (A+B)(A+B) \\ &= A(A+B) + B(A+B) \\ &= A^2 + AB + BA + B^2 \\ &= A^2 + 2AB + B^2 \end{aligned}$$

40) a)If x_0 solves $Ax = 0$, then $Ax_1 = 0$. Similarly, if x_1 solves $Ax = b$, then $Ax_1 = b$. Adding the two equations together we get

$$Ax_0 + Ax_1 = 0 + b$$

$$A(x_0 + x_1) = b$$

so that $x_0 + x_1$ also solves $Ax = b$. b)

$$Ax_1 - Ax_2 = b - b$$

$$A(x_1 - x_2) = 0$$

so $x_1 - x_2$ solve the homogeneous equation $Ax = 0$.