5.5

4) Since the inhomogeneity is of the frm xe^x, we need to a trial particular solution made up of xe^x and the parts of its derivatives that are different. Since $(xe^x)' = xe^x + e^x$, we take $y_p = Ae^x + Bxe^x$ which gives the two equations $9A + 12B = 0, 9B = 3$ considering the initial condition. The particular solution is $y_p = (-4e^x + 3xe^x)/9$.

10) Since there is duplication with the complementary function, we take $y_p = x(A\cos(3x) + B\sin(3x))$ and get that $y_p = (2x\sin(3x) - 3x\cos(3x))/6$.

34) $y(x) = \cos(x) - \sin(x) + 1/2x\sin(x)$.

5.6

2) $x(t) = 3/2\sin(2t) - \sin(3t)$.

6) Noting that $\omega_0 = \sqrt{\frac{k}{m}}$, we have the solution of the initial value problem $\frac{v_0}{\omega_0}\sin(\omega_0 t) + \frac{F_0}{2m\omega_0}t\sin(\omega_0 t)$.

12) $x(t) = -\frac{25}{87}\cos(5t) - \frac{10}{87}\sin(5t) - \frac{50}{174}e^{-3t}\cos(2t) - \frac{125}{174}e^{-3t}\sin(2t)$.

6.1

6) $\lambda = 2, v = (1,1)$; $\lambda = 3, v = (4,3)$;

12) $\lambda = 3, v = (3,2)$; $\lambda = 4, v = (5,3)$;

20) $\lambda = 1(multiplicity of 2), v_1 = (1,0,2), v_2 = (3,2,0)$; $\lambda = 2, v = (0,-2,5)$;

24) $\lambda = 1(multiplicity of 2), v_1 = (1,0,0,0), v_2 = (0,1,0,0)$;$\lambda = 3(multiplicity of 2), v_1 = (0,0,0,1), v_2 = (2,2,1,0)$;

36) If A is triangular, then $A - \lambda I$ is triangular. The determinant of a triangular matrix is the product of the diagonal elements, $(a_{11} - \lambda)(a_{22} - \lambda)...(a_{nn} - \lambda)$. Setting this equal to zero it is clear that the roots are the diagonal elements of A which are exactly the eigenvalues.

7.1

4) Let $y = x'$ and $w = y'$, then $w' = x^{(3)} = 2/tx'' - 3/t^2x' - 5x + \ln(t)/t^3$,which yields the system

$$\begin{aligned} x' &= y \\ y' &= w \\ w' &= 2/tw - 3/t^2y - 5x + \ln(t)/t^3 \end{aligned}$$

6) Let $w = x'$ and $z = y'$ so that $w' = 5x - 4y$ and $z' = -4x + 5y$,which yields the system

$$\begin{aligned} x' &= w \\ w' &= 5x - 4y \\ y' &= z \\ z' &= -4x + 5y \end{aligned}$$

14) differentiating the first equation we get the second order differential equation for x as $x'' + 100x = 0$, which gives rise to the roots $r - \pm 10i$ and the general solution $x(t) = A\cos(10t) + B\sin(10t)$. Using the x initial condition, we see that $A = 3$. Since $x' = 10y$, we see that $y = -3\sin(10t) + B\cos(10t)$. Now using the y initial condition, we see that B = 4, so that the final solution is $x(t) = 3\cos(10t) + 4\sin(10t)$ and $y(t) = 4\cos(10t) - 3\sin(10t)$.

20) Differentiating the first equation we get the second order equation for x as $x'' - 6x' + 9x = 0$, which yields the repeated root $r = 3$ and teh general solution $x(t) = Ae^{3t} + Bte^{3t}$ and then for y, $y(t) = (3A + B + 3Bt)e^{3t}$.

24)