In this note we show that our reconstruction method will, at every time step generate at least one solution (out of two) with at least one of the conductances taking a negative value.

We start by assuming that all solutions for the conductances $g_1$, $g_2$, and $g_3$ at the $i$th time step are positive real values and show that this leads to a contradiction, implying that at least one of them must be negative. For convenience define $\Delta_1 = (\mu_{V_i} - E_1)$, $\Delta_2 = (\mu_{V_i} - E_2)$, and $\Delta_3 = (\mu_{V_i} - E_3)$. One can write the equation relating the variances of $V_m$ at successive time steps as

$$\sigma_{V_{i+1}}^2 = \sigma_{V_i}^2 \left( 1 - \frac{\Delta t}{C} (g_L + g_1 + g_2 + g_3) \right)^2 + \frac{\Delta t}{C^2} \sum_{j=1}^{3} g_j^2 \sigma_{\xi_j}^2 (\sigma_{V_i}^2 + \Delta_j^2)$$

(same as equation (18) in the manuscript).

For $\Delta t$ sufficiently small one has that $\sigma_{V_i}^2 \approx \sigma_{V_{i+1}}^2$ allowing replacement of (1) by

$$\sigma_{V_i}^2 = \sigma_{V_i}^2 \left( 1 - \frac{\Delta t}{C} (g_L + g_1 + g_2 + g_3) \right)^2 + \frac{\Delta t}{C^2} \sum_{j=1}^{3} g_j^2 \sigma_{\xi_j}^2 (\sigma_{V_i}^2 + \Delta_j^2)$$

Expanding (2) gives

$$\sigma_{V_i}^2 = \sigma_{V_i}^2 - \frac{2 \Delta t}{C} \sigma_{V_i}^2 (g_L + g_1 + g_2 + g_3) + \frac{\Delta t^2}{C^2} \sigma_{V_i}^2 (g_L + g_1 + g_2 + g_3)^2 + \frac{\Delta t}{C^2} \sum_{j=1}^{3} g_j^2 \sigma_{\xi_j}^2 (\sigma_{V_i}^2 + \Delta_j^2)$$

The common $\sigma_{V_i}^2$ terms may be cancelled on both sides. We may also ignore the $O(\Delta t^2)$ term for small $\Delta t$. Multiplying the resulting equation by $C^2/\Delta t$ gives

$$0 = -2C \sigma_{V_i}^2 (g_L + g_1 + g_2 + g_3) + \sum_{j=1}^{3} g_j^2 \sigma_{\xi_j}^2 (\sigma_{V_i}^2 + \Delta_j^2)$$

We may now express (4) as a quadratic equation in terms of $g_1$,

$$ag_1^2 + bg_1 + c = 0$$

where

$$a = \sigma_{\xi_i}^2 (\sigma_{V_i}^2 + \Delta_1^2)$$

$$b = -2C \sigma_{V_i}^2$$

$$c = g_2^2 \sigma_{\xi_2}^2 (\sigma_{V_i}^2 + \Delta_2^2) + g_3^2 \sigma_{\xi_3}^2 (\sigma_{V_i}^2 + \Delta_3^2) - 2C \sigma_{V_i}^2 (g_L + g_2 + g_3).$$

If both solutions for $g_1$ are positive real values, then by the quadratic formula,

$$g_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
it follows that \(-4ac < 0\). Since (6) shows that \(a\) is always positive, this is equivalent to the condition that \(c > 0\). By (5) this implies that \(ag_1^2 + bg_1 < 0\) or that
\[
\sigma_{\xi_1}^2(\sigma_{\xi_1}^2 + \Delta_1^2)g_1^2 < 2C\sigma_{\xi_1}^2g_1 < 0
\]  
(9)

Symmetric arguments can be made for \(g_2\) and \(g_3\) assuming all solutions are positive real values. These yield,
\[
\begin{align*}
\sigma_{\xi_2}^2(\sigma_{\xi_2}^2 + \Delta_2^2)g_2^2 &< 2C\sigma_{\xi_2}^2g_2 < 0 \quad (10) \\
\sigma_{\xi_3}^2(\sigma_{\xi_3}^2 + \Delta_3^2)g_3^2 &< 2C\sigma_{\xi_3}^2g_3 < 0 \quad (11)
\end{align*}
\]

Equation (4) may now be rewritten as
\[
2C\sigma_{\xi_1}^2g_L = -2C\sigma_{\xi_1}^2(g_1 + g_2 + g_3) + \sum_{j=1}^{3} g_j^2\sigma_{\xi_j}^2(\sigma_{\xi_j}^2 + \Delta_j^2).
\]  
(12)

Adding the inequalities (9)-(11) shows that the right hand side of (12) is negative. However, since \(g_L > 0\), it must be the case that the left hand side of (12) is positive. This contradiction implies that at least one of the solutions for \(g_1\), \(g_2\), or \(g_3\) must be negative and extraneous. Moreover, the two associated solutions for the other conductances will also be extraneous.