

Figure 9.4. Graphical illustration of the range expansion of sea otters. (Figure 1 of Lubina and Levin [110], reprinted with permission from Chicago University Press.)

### 9.4 Physiology

## Project 19: Pupil Control System

The pupil is the opening in the middle of the eye through which light enters the eye. In many animals, including humans, involuntary contraction and dilation of the pupil regulates the intensity of light entering the eye. The pupil will contract under bright light conditions, while it will dilate under low light conditions.

Now suppose that you shine a tiny spot of light onto the eye, always in the same location, and that the spot initially is on the edge of the pupil. At first, the pupil will contract in response to the spot of light. After the contraction, the light no longer enters the pupil, so it will dilate. After the dilation, the light again enters the pupil, so it will contract again, and so forth. An oscillation has been generated, for example, as shown in Figure 9.5.

Develop a model to reproduce the phenomenon. Begin by developing a model of the light intensity as a function of the pupil radius. Then add in negative feedback. Can you obtain oscillations? Investigate the incorporation of a delay representing the time it takes for the eye to respond to a change in light intensity.

Table 9.10. Experimental data on the range expansion of sea otters. (Table 1 of Lubina and Levin [110], reprinted with permission from Chicago University Press.)

| Year | ExTENT OF Range Increase |  | Estimated Total Range | Population Size |
| :---: | :---: | :---: | :---: | :---: |
|  | North | South |  |  |
| 1914 | ? | ? | (11) | (50) |
| 1938 | 11 | (21) | 43 | 310 |
| 1947 | 8 | 23 | 74 | 530 |
| 1950 | 2 | 13 | 89 | 660 |
| 1955 | 3 | 16 | 108 | 800 |
| 1957 | 11 | 6 | 125 | 880 |
| 1959 | 6 | 6 | 137 | 1050 |
| 1963 | 5 | 10 | 152 | 1190 |
| 1966 | 0 | 6 | 158 | 1260 |
| 1969 | 6 | 13 | 177 | 1390 |
| 1972 | 0 | 15 | 192 | 1530 |
| 1973 | 23 | 29 | 244 | 1720 |
| 1974 | 6 | 5 | 255 | 1730 |
| 1975 | 8 | 0 | 263 | ? |
| 1976 | 10 | 6 | 279 | 1789 |
| 1977 | 8 | 6 | 293 | ? |
| 1978 | 0 | 0 | 293 | ? |
| 1979 | 0 | 6 | 299 | (1443) |
| 1980 | 0 | 13 | 312 | ? |
| 1982 | 0 | 0 | 312 | 1338 |
| 1983 | 26 | 15 | 353 | 1226 |
| 1984 | 0 | 0 | 353 | 1203 |
| 1986 | ? | ? | ? | 1400 |

Note. - The extent of the range is determined by the linear distance along the coastine between the outermost main raft of otters at the population boundaries. Point Sur was used as the location of the division between the northern and southern populations. The total estimated population size was based on aerial and shore counts. Parentheses indicate that the estimate was considered unreliable; a question mark means that no estimate was made.

Sources.-E. Ebert, pers. comm.; Riedman and Estes, MS; Estes, unpubl. data.

## Project 20: Modeling of Heart Beats

In this project (inspired by Chapter 1.13 in [1]), you are asked to investigate the production of heart beats.

A mathematician's view of the apparatus for beating of the heart is shown in Figure 9.6. The sinoatrial (SA) node is the pacemaker. Its function is to send signals at regular intervals


Figure 9.5. (a) Technique used for pupil stimulation. Light here is focused on the border of iris and pupil. Small movements of the iris result in large changes in light intensity of the retina. (b) Example of spontaneous high gain oscillations in pupil area obtained with constant light stimulus using high gain operating condition illustrated in (a). (Figures 11 and 12 in Section II of Stark [149], reprinted with permission from Kluwer/Plenum Press.)
to the atrioventrical (AV) node. Upon receipt of a signal from the SA node, the AV node checks the condition of the heart and decides whether to tell the heart to contract or not.

For a simple model of the heart, it is sufficient to describe the behavior of the AV node. The AV node uses an electrical potential to keep track of the condition of the heart. In particular, this potential decreases exponentially during the time between signals from the SA node. When the AV node receives a signal from the SA node, one of two things happens. If the potential is too high, it means that the heart is not yet ready to contract again. and the AV node ignores the signal. Otherwise, the AV node tells the heart to contract. The contraction of the heart causes the potential of the AV node to increase (for simplicity, you may assume that the increase is a constant).

Develop a model describing the electrical potential of the AV node. Under what conditions does your model produce regular heart beats?

Investigate the production of irregular beating patterns by modifying parameters in your model. Two patterns of clinical interest are second-degree block and the Wenckebach


Figure 9.6. A schematic of the heart beat control mechanism. Figure adapted from Adler [1].
phenomenon. Second-degree block refers to the situation in which the heart skips every other beat (i.e., the AV node blocks every other signal from the SA node). The Wenckebach phenomenon refers to the situation in which the heart skips a beat every now and then, while it beats normally most of the time. Can your model produce other beating patterns?

## Project 21: Ocular Dominance Columns

Visual information is transmitted via the optic nerve to the visual cortex. Scientists studying the visual cortex of cats and monkeys discovered columns (bands or stripes) of neurons that selectively respond to visual information from one eye or the other. The bands are interlaced, as shown in Figure 9.7.

Hubel et al. [94] suggested that the columns are formed through a competition process during the first several months after birth. Neurons in the visual cortex have a number of synapses receiving inputs from the eyes. A synapse is associated either with the right eye or the left eye. Initially, all neurons are binocular, that is, it has both right- and lefteye synapses, and the synapses are intermixed randomly. During development, synapses can switch allegiance from one eye to the other, as a result of competition. Swindale [153] demonstrated that ocular dominance patterns can be generated by assuming that interactions between right- and left-eye synapses follow two simple rules:
(1) Local interactions (within a region $200 \mu \mathrm{~m}$ in diameter) are stimulatory (for example, in a region where right-eye synapses dominate, there will be an increase in the number of right-eye synapses at the expense of left-eye synapses);
(2) Interactions over larger distances ( $200-600 \mu \mathrm{~m}$ ) between opposite-eye synapses are inhibitory (for example, in an annular ring surrounding a region where right-eye


Figure 9.9. Shells of Olivia and Conus species.

## Project 24: Mollusk Patterns

Many mollusks show very interesting patterns on their shells. Figure 9.9 shows examples of Olivia spec. and Conus spec. Since these shells grow gradually on the outer edge only and since the patterns do not change later, it can be seen as the time record of a one-dimensional pattern-producing system.

These shell patterns are very similar to some patterns produced by some simple Wolfram automata (discussed in Section 6.1.1). Therefore, they might be modeled with cellular automata as done by Kusch and Markus [103].

Find rules for a one-dimensional cellular automaton that produces patterns as shown in Figure 9.9. You may also look for pictures of other Olivia and Conus species and reproduce their patterns. If you look closely at these pictures, you see that real shell patterns are never as perfect as patterns produced by simulations. Introduce stochasticity in your automaton to generate more realistic patterns.

Please note that if two patterns look alike, it does not necessarily mean that they are produced by the same mechanism. You may compare your model with the reaction-diffusion models in [117].

## Project 25: Run-Bike-Fun

A "Run-Bike-Fun" sports event takes place every year in a small university town in Germany. Each participating team consists of two people. Both people have to complete a 15 km course through a combination of running and cycling. Each team has one bicycle. Only one person is allowed to ride the bicycle at any one time, but team members can switch between running and cycling as often as they wish. The first team with both partners at the finish line wins.

At the beginning of the race, one person starts riding the bicycle, and the other starts running. After some time, the cyclist gets off the bicycle, puts it down, and starts running. When the other runner reaches the bicycle, he/she picks it up and starts cycling.

What is the optimal switching strategy? At which locations along the course should the switch(es) occur?

You may wish to begin by assuming that it takes no time to get on/off the bicycle, that both team members are $x$ times faster at cycling compared to running, and that people run/cycle with constant velocity. Based on your own experience, estimate the value of $x$. When/where should you switch?

In reality, people get tired. How might you describe that? Would you use the same description for running and cycling? How does this affect the optimal strategy? Also, switching between cycling and running takes time. How does this affect the optimal strategy? What if two people with different abilities form a team?

