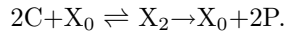


Math 5110/6830
Instructor: Alla Borisyuk
Homework 9
Due: November 14

1. Consider the following reaction:



Repeat the same steps we did in class for a similar system to show that the dynamics of these reactions can be reduced to

$$\frac{dc}{dt} = \frac{-K_{max}c^2}{k_n + c^2}.$$

What are the values of K_{max} and k_n in terms of the original parameters (reaction rates and the total number of receptors)? (Hint: write differential equations for all components, use constancy of the receptor number to reduce the number of equations, and use quasi steady state approximation to further reduce the system)

2. Study the system

$$\begin{aligned} \frac{dc}{dt} &= -k_1rc + (k-1 + k_1c)x_1, \\ \frac{dx_1}{dt} &= -k_1rc + (k-1 + k_2 + k_1c)x_1 \end{aligned}$$

by phase-plane methods. Namely:

a) Show that nullclines have the form

$$\begin{aligned} \dot{c} = 0 & \quad \text{when} \quad x_1 = \frac{Kc}{a+c}, \\ \dot{x}_1 = 0 & \quad \text{when} \quad x_1 = \frac{Kc}{b+c}. \end{aligned}$$

Identify K, a, b in terms of the original parameters. Which is larger, a or b ?

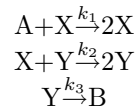
b) Sketch the nullclines. How many points of intersection are there?

c) Sketch the direction field along the nullclines and along the c and x_1 axes.

d) Draw a trajectory beginning at the state in which all receptors are unoccupied and the initial nutrient concentration is c_0 . What is the eventual outcome?

e) Explain your result in terms of the original cellular process.

3. The following chemical reaction mechanism was studied by Lotka:



Assume that A and B are kept at constant concentration.

a) Write a set of equations for the concentrations of X and Y using the law of mass action.

b) Study the steady states of the system and their stability. What can you say about oscillatory solutions?