

**Math 5110/6830**  
**Instructor: Alla Borisjuk**  
**Homework 8.1**  
**Due: November 7**

1. Show that the claim made in class is true:  $\det J = 0$  can only happen at the “knee” of  $I_{ss}(V)$ , i.e. where  $\frac{dI_{ss}}{dV} = 0$ , and, consequently cannot happen when  $I_{ss}(V)$  is monotone.

2. Using the functions and parameter values given below for the Morris-Lecar model, do the following in Matlab:

a) Plot  $I_{ss}(V)$  to see that it is indeed monotone.

b) Plot nullclines for  $I_{app} = 0$ .

c) Plot nullclines for  $I_{app} = -50$  ON THE SAME GRAPH! as b)

d) Using the graph from c) describe what will happen if  $I_{app}$  is held at -50 for a long time, and then turned off ( $I_{app} = 0$ ) (Hint: sketch the direction field after the current is turned off and draw the solution trajectory). This phenomenon is called a post-inhibitory rebound (or anodal break excitation). It demonstrates that cells can be made to spike by applying negative current too! (not just by a positive “kick” (excitable state) or positive applied current (through the Hopf bifurcation), that we saw in class)

e) **(Extra credit)** What is the biophysical requirement for observing the phenomenon from d)? (Hint: think about the properties of the potassium channels)

Standard set of functions and parameters:

$$m_{\infty}(V) = .5 * (1 + \tanh((V - V_1)/V_2)),$$

$$w_{\infty}(V) = .5 * (1 + \tanh((V - V_3)/V_4)),$$

$$\tau_w(V) = 1 / \cosh((V - V_3)/(2 * V_4)),$$

$$\bar{g}_{Ca} = 4,$$

$$V_{Ca} = 120,$$

$$\bar{g}_K = 8,$$

$$V_K = -84,$$

$$g_L = 2,$$

$$V_L = -60,$$

$$\phi = 0.04,$$

$$V_1 = -1.2,$$

$$V_2 = 18,$$

$$V_3 = 2,$$

$$V_4 = 30.$$

3. Now use  $V_3 = 12$ ,  $V_4 = 17$ ,  $\phi = .06666667$ .

a) Study the stability of the steady states (analytically or graphically)

- b)** Plot the nullclines for several values of  $I_{app}$ . What bifurcations, do you think, will happen as  $I_{app}$  is increased or decreased away from zero?
- c)** Sketch the bifurcation diagram (steady states only). **Extra credit:** indicate stability