Math 5110/6830
Instructor: Alla Borisyuk
Homework 7.2
Due: October 31

1. (St: ch3) For each of the following exercises do:
(i) sketch all the qualitatively different vector fields that occur as μ is varied.
(ii) Find the bifurcation value of μ graphically.
(iii) Find the bifurcation value of μ analytically, if possible.
(iv) Identify the bifurcation.
(v) Sketch the bifurcation diagram with μ as a parameter.

a) $\dot{x} = \mu x + 4x^3$

b) $\dot{x} = x + \frac{\mu x}{1+x^2}$

c) $\dot{x} = \mu x - \frac{x}{1+x}$

2. As we mentioned in class, the "signature" of the Hopf bifurcation is that the eigenvalues at the bifurcation value are purely imaginary. By doing the linearization at the origin and computing the eigenvalues, show that the system $\dot{x} = -y + \mu x + xy^2$, $\dot{y} = x + \mu y - x^2$ undergoes a Hopf bifurcation at $\mu = 0$.