

Math 5110/6830
Instructor: Alla Borisyyuk
Homework 5.2
Due: October 17

1. Use linear stability analysis to classify the fixed points of the following systems. If the linear stability analysis fails, use a graphical argument.

a) $\dot{x} = 1 - e^{-x^2}$

b) $\dot{x} = \ln x$

c) $\dot{x} = x(1-x)(2-x)$

d) $\dot{x} = ax - x^3$. Discuss cases when a is positive, negative or zero.

2. A particle travels on the half-line $x \geq 0$ with a velocity given by $\dot{x} = -x^c$, where c is real and constant.

a) Find all values of c such that the origin $x = 0$ is a stable fixed point

b) Now assume that c is chosen so that $x = 0$ is a fixed point. How long does it take the particle to travel from $x = 1$ to $x = 0$, as a function of c ?

3. Show that the initial value problem $\dot{x} = x^{1/3}$, $x(0) = 0$ has an infinite number of solutions. (Hint: Construct a solution that stays at $x = 0$ for some arbitrary time t_0 , after which it takes off.)